NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2642

APPLICATION OF LINEAR ANALYSIS TO AN EXPERIMENTAL INVESTIGATION OF A TURBOJET ENGINE WITH PROPORTIONAL SPEED CONTROL

By Marcel Dandois and David Novik

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

Washington
February 1952

1A

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2642

APPLICATION OF LINEAR ANALYSIS TO AN EXPERIMENTAL INVESTIGATION OF A TURBOJET ENGINE WITH PROPORTIONAL SPEED CONTROL

By Marcel Dandois and David Novik

SUMMARY

Results of an analytical and a sea-level experimental investigation of a turbojet engine with proportional speed control are presented. Linear analysis and description of the engine as a first-order linear system proved adequate for analytical prediction of the response and the stability of the controlled engine, although instability calculations were found to be much more critical than response calculations.

On the basis of a compromise between speed of response and oscillations, an optimum loop gain was found. Increased loop gain increased the speed of response and decreased the speed error but ultimately led to instability characterized by an essentially constant frequency and constant amplitude oscillation. Operation near the limits of stability required a decrease in control gain with decreasing engine speed.

INTRODUCTION

A current approach to synthesis of control systems for turbojet engines involves the techniques of linear analysis and synthesis described in references 1 and 2 and their bibliographies. This approach is predicated upon a knowledge of the dynamic characteristics of the various components of the controlled system. Accordingly, considerable effort has been expended in obtaining the dynamic characteristics of gas-turbine engines from experimental and analytical studies (references 3 to 6). These investigations of gas-turbine engine dynamics indicate that the engines may be considered linear for substantial excursions from steady-state equilibrium points and that within the limits of experimental accuracy the engines appear to be first-order systems.

The adequacy of the linear techniques and data discussed in the references may be ascertained by comparison of analytical results based on these techniques and actual experimental data. An experimental investigation of a turbojet engine controlled by various types of control system has been initiated at the NACA Lewis laboratory to provide experimental data for this comparison and simultaneously to determine the characteristics of the different types of control system utilized.

The results obtained from the initial phase of the experimental program, a sea-level experimental investigation of proportional speed control, are presented herein. Proportional speed control was selected as the initial phase of the project because engine speed is the controlled variable in most engine control systems now in use and because proportional control is a dominant factor in virtually every type of control system. The results obtained from this investigation should be applicable, at least in part, to studies of any specific control system for turbojet engines.

The use of a purely proportional control consisting only of an amplification factor would not contribute any fundamental change to the dynamics of the engine and if the engine is considered as a first-order system instability could not be obtained and examined. In order to introduce the possibility of instability, the control system was permitted to contribute several lags to the controlled engine system in addition to the purely proportional control element. A nonlinearity other than the engine was also included so that the adequacy of linear analysis was severely tested.

APPARATUS

Engine

The turbojet engine used in this investigation was of the axial-flow type having a twelve-stage compressor, a single-annulus combustion chamber, and a two-stage turbine. Experimentally determined steady-state performance curves of the engine are shown in figure 1.

Control System

The engine control system consisted essentially of an electronic amplifier with variable gain for amplification of an error signal and an oil system with a pressure regulating servovalve positioned by the output voltage of the electronic amplifier, such that a given pressure exists for each output voltage. The control oil pressure acts on the servopiston of a motor-driven constant-pressure fuel pump (reference 7) which supplies a proportional fuel pressure to the engine. For convenience of analysis and data presentation, the oil system is considered as part of the fuel system and therefore the electronic amplifier can be considered as the proportional control element. A schematic drawing of the controlled engine system is shown in figure 2. Lags in the system include the oil system, the fuel system, and the RLC (resistance-inductance-capacitance) filter, as well as the engine.

Instrumentation

Recorder. - All transient data were recorded on a multichannel recording oscillograph. The oscillograph elements had a sensitivity of 1-inch deflection per 0.43 millivolt and were damped to an effective time constant of approximately 0.005 second.

Engine speed. - Transients in engine speed were measured by recording the output of a small tachometer generator on the recording oscillograph. The high frequency ripple from the generator was filtered out of the speed measuring circuit with an RC (resistance-capacitance) circuit filter. In the closed loop (fig. 2), where engine speed is fed back into the loop and where phase shift and amplitude attenuation become important, an RLC filter was utilized. The frequency response characteristics of both filters are shown in figure 3.

Pressures. - Transients in fuel pressure and control oil pressure were measured with bridge-type strain-gage pressure pickups energized by a 3000 cycle carrier system. The apparent time constant of the pressure measuring circuits was 0.02 second.

ENGINE DYNAMICS

Engine dynamics were obtained by observing the response of the engine variables to approximate step inputs in fuel pressure and to sinusoidal fuel pressure changes of variable frequency. These tests were run with an open loop system, that is, engine speed feedback was not used and the electronic control was omitted.

Typical data showing the response of the uncontrolled engine are shown in figure 4. Variables other than control oil pressure, fuel pressure, and engine speed were obtained for further analysis but are not discussed in this report. From these data, the corrected time constant of engine speed with respect to fuel pressure was obtained and is shown in figure 5 as a function of final corrected engine speed. The time constants shown were obtained from the frequency response characteristics of the variables as indicated by harmonic analyses of the data obtained with digital computer machines and by the response to sinusoidal inputs. Frequency response characteristics of the engine, corrected for the filter characteristics, are shown in figure 6 for three engine speeds. Frequency response characteristics of the fuel system, including the oil system, were obtained from sinusoidal inputs and are shown in figure 7 for three engine speeds.

RESULTS AND DISCUSSION

Effect of Loop Gain on Response and Error

Response. - The controlled engine system shown schematically in figure 2 was operated over a range of proportional control settings (gain settings) for several conditions of engine speed. Transients were induced by means of a step input in voltage (N_s of fig. 2) into the electronic control and the responses of the variables were recorded. Data typical of the responses obtained are shown in figure 8. Traces shown for speed error N_s-N also represent the negative of engine speed inasmuch as the speed setting N_s is constant after the initial step transient.

The loop gains shown in the data are defined as the product of the gains at zero frequency of all the components in the closed loop (fig. 2) such that:

$$\text{loop gain} = \underset{\substack{\text{(control}\\\text{amplifier)}}}{\left(\frac{\text{volts}}{\text{volt}}\right)} \underset{\substack{\text{(oil}\\\text{system)}}}{\left(\frac{\text{lb/sq in.}}{\text{volt}}\right)} \underset{\substack{\text{(fuel}\\\text{pump)}}}{\left(\frac{\text{lb/sq in.}}{\text{lb/sq in.}}\right)} \underset{\text{(engine)}}{\left(\frac{\text{rpm}}{\text{lb/sq in.}}\right)} \underset{\text{(tachometer)}}{\left(\frac{\text{volts}}{\text{rpm}}\right)} \underset{\substack{\text{(tachometer}\\\text{amplifier)}}}{\left(\frac{\text{volts}}{\text{volt}}\right)}$$

$$= \text{dimensionless quantity}$$

These loop gains are calculated from step input N_s data from the expression

$$\text{loop gain} = \frac{\text{initial error}}{\text{final error}} - 1 \qquad (1)$$

This equation is valid for all controlled systems that do not contain an integral term because the final error in such systems is zero.

In order to show the comparative effect of different loop gains on the speed of response of the controlled engine, an arbitrary indication of the initial speed of response was selected rather than the time required for attainment of equilibrium. This selection was made because the time required for equilibrium to be reached does not necessarily decrease with increasing loop gain, inasmuch as the oscillations resulting from an underdamped system at high gain may take longer to damp out as the gain is increased. The arbitrary criterion of response time was taken as the time required for the engine speed to first attain 63 percent of its ultimate equilibrium speed and may be compared with the time constant of the uncontrolled engine. The effect of increasing loop gain on the response time of the controlled engine (time required for engine to attain 63 percent of equilibrium speed) is shown in figure 9 for three engine speeds. Figure 9 indicates that the controlled engine responds more rapidly as the loop gain increases, although the improvement in response becomes less pronounced at the higher values of loop gain. The response of the controlled engine at 84 percent maximum engine speed is faster than at 90 percent maximum engine speed because the time constant of the engine increased slightly as the speed was increased above 84 percent maximum engine speed.

The effect of lags and higher-order systems on response is shown by a comparison of the experimental data at 84 percent maximum engine speed with the theoretical response of a first-order system (fig. 10). The first-order curve is based only on the engine time constant at 84 percent maximum engine speed and all other components are neglected. This curve was calculated from the operational equation

$$\frac{N}{N_s}(p) = \frac{\dfrac{G}{1 + \tau_1 p}}{1 + \dfrac{G}{1 + \tau_1 p}} \tag{2}$$

where τ_1 is the engine time constant at 84 percent of maximum engine speed (approximately 1.2 sec).

The comparison of the experimental data with the theoretical first-order curve indicates that the response time increases for higher-order systems and that the improvement in response with increased gain becomes more marginal for a system with lags.

The response of a theoretical second-order system, the curve of which is shown in figure 10, is calculated from the following operational equation:

$$\frac{N}{N_s}(p) = \frac{\dfrac{G}{(1 + \tau_1 p)(1 + \tau_2 p)}}{1 + \dfrac{G}{(1 + \tau_1 p)(1 + \tau_2 p)}} \tag{3}$$

where, again,

τ_1 engine time constant

τ_2 first-order approximation of oil and fuel systems obtained from 63-percent point of response of fuel pressure to a step input (0.28 sec, fig. 4(b))

The higher-order system of the actual controlled engine has been simplified analytically by considering it as only a second-order system. The excellent agreement between the experimental data obtained and the calculated second-order curve indicates that many simplifying assumptions may be made in calculating response without causing appreciable deviations from the experimentally obtained response of a complex physical system.

Because the system becomes more and more underdamped as the loop gain increases (fig. 8), the magnitude and duration of the oscillations at high gain tend to negate the advantages of the improved initial response. A criterion is therefore required that will compromise between the fast initial response at high gain and the oscillations incurred. Such a criterion may reasonably be based on the time integral of the square of the speed error produced by a given disturbance. The premise is made that the best compromise between response and oscillations will occur when this integral is a minimum. Mathematically, this relation may be expressed as

$$\int_{t_i}^{t_f} (N_t - N_f)^2 \, dt = \text{minimum}$$

The speed error is squared in order to give a positive value to the integral when the oscillations result in negative values of engine speed with reference to the final engine speed (reference 8).

Values of this integral plotted against loop gain for three engine speeds and a small pictorial representation of the meaning of the integral are shown in figure 11. For the specific system used in this investigation, the values of the integral tend toward a minimum which may be taken to occur at a loop gain of about 8. According to figure 9, the region of this minimum also corresponds to loop gains at which the improvement in response with increasing gain may be considered marginal.

Figures 9 and 11 indicate that a proportional control setting that would give a loop gain of about 8 would result in optimum proportional control on the basis of a compromise between initial response and minimum oscillation. For systems with less lag, this optimum gain would be slightly higher.

Error. - An obvious characteristic of proportional speed control is that as the gain is increased, the engine speed will more closely approach the speed setting, inasmuch as a smaller error N_s-N will be required to support the engine speed at a given value. This characteristic is relatively unimportant when the desirability of arriving at an engine speed close to the speed setting is considered, because the speed setting throttle can always be calibrated or adjusted with allowance for the speed error. However, if the effects of disturbances other than those initiated from the control lever are considered, such as those from a plugged fuel nozzle or large change in airplane angle of attack, the relation between gain and error assumes greater importance. Equilibrium conditions more closely corresponding to the original conditions will be restored and the response will be faster if the gain is high because the entire loop is more sensitive at high gain and only a relatively small change in error results in a comparatively large corrective signal to the engine.

Data typical of the engine response to a disturbance other than a change in control setting are shown in figure 12. These data were obtained by closing a solenoid valve in one of two fuel lines in parallel. Because the fuel pump was a constant pressure pump, the increased resistance to flow resulted in a decreased fuel flow from the pump.

The data presented show the sudden decrease in fuel pressure followed by its gradual increase as the control system attempted to restore the original equilibrium conditions. The engine speed following the fuel flow disturbance responds more rapidly and more nearly returns to the original value for the setting of higher gain. A plot of the difference between initial and final engine speeds for various values of loop gain is shown in figure 13 for a constant value of disturbance in the fuel system equivalent to a 5 percent maximum engine speed change in the uncontrolled engine. The calculated curve was obtained through the use of equation (1).

Figures 12 and 13 show that as the gain is increased the change in error due to random disturbances decreases. This change in error is the difference between the initial and final speeds as plotted on figure 13 and is actually a change in engine speed only, inasmuch as the speed setting N_s remains fixed.

Instability

Occurrence and nature of instability. - As the gain settings were increased, instability was eventually attained at each engine speed. Typical instability data are shown in figure 14.

Controlled engine instability manifested itself as an essentially constant frequency and constant amplitude oscillation. The sinusoidal characteristic of this instability is indicative of the system's linearity, whereas the fact that the amplitude remains approximately constant is indicative of limits and other nonlinearities within the system. As an ideal linear system would oscillate with ever increasing amplitude, the presence of nonlinearities has apparently stabilized the system to a condition of steady oscillations.

The oscillations illustrated in figure 14 were obtained at approximately the lowest possible loop gains at which instability could be attained. At successively higher engine speeds, the frequency of oscillations increased and the magnitude decreased from about 2 to 0.5 percent of maximum engine speed. Further increase in loop gain beyond the gain at which instability first occurred resulted in a slightly increased amplitude and frequency of oscillation (these data are not shown). All oscillations obtained represented unfavorable conditions of engine operation.

Effect of increased phase shift in system. - According to linear theory, instability occurs when a loop gain of at least unity exists at the frequency corresponding to a total phase shift of 180° between input (error) and output (speed) of a closed loop system (references 1 and 2). In order to indicate the effect of phase lag on instability, the total phase shift through the system therefore was increased by substituting the large phase lag RC filter for the RLC filter normally used (fig. 3). Instability data shown in figure 15 were obtained.

As a result of the increase in phase lag, the maximum allowable loop gain prior to attainment of instability was considerably reduced at all engine speeds. A comparison of the loop gains at instability for the original system (with RLC filter) and the system with increased phase lag (RC filter) is shown in figure 16 over a range of engine speeds.

Comparison of figure 14 with figure 15 shows that at the same engine speeds, the original system oscillated at a higher frequency than the system with increased phase lag. The greater allowable loop gain prior to instability and the higher frequency of oscillation agrees with linear instability theory in that reduction of phase shift may be expected to displace the 180° phase lag point to correspond to a higher frequency and lower amplitude at a higher loop gain.

Prediction of loop gain at instability. - Although the type of instability obtained in this investigation exhibited certain nonlinear characteristics (see control oil trace, fig. 15(a)), the linearity apparent from the data indicated that reasonable estimates of the loop gains corresponding to instability could be made. The method of calculation consisted in cascading the transfer functions of the components within the closed loop system, as represented by their frequency response characteristics. By multiplying the amplitude ratios and adding the phase lags of the fuel system, the engine, and the RLC filter, the overall frequency response of the system was obtained for each specific engine speed.

The amplitude ratios of each component and of the closed loop system were taken as unity at zero frequency. The calculated gain at instability was therefore the factor required to increase the amplitude ratio of the closed loop system at 180° phase lag to a value of unity.

The frequency response characteristics of the components are presented in figures 3 (RLC filter), 6, and 7. The cascaded frequency response and calculated gain at instability for the instabilities of figure 14 are shown in figure 17. A comparison of the calculated and experimental data appears in the following table:

Engine speed (percent maximum)	Measured loop gain at instability	Calculated loop gain at instability	Measured frequency at instability (radians/sec)	Calculated frequency at instability
64	12.2	10.0	4.3	3.7
80	10.0	12.0	7.5	6.4
88	9.6	12.5	9.8	7.5

Comparison of the calculated and actual gains at instability shows that despite nonlinearities within the physical system, the loop gain at instability can be reasonably predicted from a knowledge of component dynamics. Frequency response data for all the components in the system were utilized for the calculations, however, and the polar plots of figure 17 show that small discrepancies in phase angles could result in large changes in the calculated loop gain at instability. Therefore the linear analysis used in the instability calculations is adequate if sufficient frequency response data for every component in the system are available. This conclusion constrasts with the fact that response time can be calculated quite accurately despite rather broad simplifying assumptions as discussed previously. An example of the difference between data required for instability and response calculations is that instability would not have been obtained at any value of loop gain for the second-order approximation used successfully in the calculation of response.

Maximum Allowable Control Gain

The variation of loop gain with engine speed limited by instability is shown in figure 16. For both systems shown the loop gain at instability decreased with increasing engine speed. This trend, however, is not directly indicative of the required variation in control gain setting (which may be considered as the independent variable with respect to loop gain adjustment) because of the large variation of engine gain and fuel system gain with engine speed (fig. 18).

If lines of constant control gain are added to figure 16, the shape of these curves will reflect the change in engine and fuel system gain with engine speed as shown in figure 19. At a constant control gain, deceleration from a high speed to a low speed could result in a transition from stable to unstable operation. For example, if operation is initially assumed at 92 percent maximum engine speed and a control gain of 0.2, a deceleration at constant control gain would result in instability at about 80 percent maximum engine speed because the engine gain and therefore the loop gain increases as the speed decreases. In order to operate near maximum loop gain at all engine speeds, the control gain must be adjusted so as to decrease with decreasing engine speeds, a trend opposite to that exhibited by the loop gain.

Numbers shown on figure 19 for values of control gain are not quantitatively significant because they include a lumping of the tachometer generator gain, tachometer generator output amplifier gain, and electronic control gain.

Mathematical Representation of Engine

The correspondence of the frequency response characteristics of the engine with theoretical first-order systems may be seen from the phase against amplitude plots of figure 6. Deviations are apparent in the middle region of phase shift but at phase shifts above 80° the discrepancies are negligible. Inasmuch as instability occurred at engine phase shifts greater than 80°, instability calculations based on theoretical first-order curves would have given the same results at those obtained from actual data. First-order linear representation of the engine therefore appears adequate for estimation of controlled engine characteristics, including prediction of the loop gain necessary to produce engine instability.

SUMMARY OF RESULTS

Results from an analytical and experimental investigation of a turbojet engine with proportional speed control were presented and discussed. Description of the engine as a first-order linear system and linear analysis based primarily on frequency response techniques proved adequate for calculation of response and instability despite certain nonlinearities in the controlled engine system. Whereas broad simplifying assumptions were permissible in calculations of response, the predictions of instability required accurate knowledge of the frequency response characteristics of every component in the system.

An optimum loop gain, based on a compromise between speed of response and oscillations, was found to exist, and for the specific system investigated, occurred at a loop gain of about 8. At higher loop gains, instability characterized by an essentially constant frequency and constant amplitude oscillation was attained. The loop gains at which instability occurred decreased with increasing engine speed. Because of large changes in engine gain it was, nevertheless, necessary to decrease the control gain with decreasing engine speed in order to permit stable operation.

As expected, the effect of increasing gain resulted in faster response, more overshoot and oscillations, and more satisfactory restoration of equilibrium for a random disturbance. Increased phase lag in the system decreased the permissible loop gain limited by instability and reduced the improvement of increased gain on response.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, November 14, 1951

APPENDIX - SYMBOLS

The following symbols are used in this report:

C	capacitance
G	loop gain
L	inductance
N	engine speed
N_s	engine speed setting
N_t	engine speed at time t
N_f	final engine speed
R	resistance
t	time
t_f	final time
t_i	initial time
T	temperature
W_a	air flow
$\dot{W}_f$	fuel flow
δ	pressure correction, ratio of ambient pressure to sea-level pressure
θ	temperature correction, ratio of ambient temperature to sea-level temperature
τ	time constant

REFERENCES

1. Brown, Gordon S., and Campbell, Donald P.: Principles of Servomechanisms. John Wiley & Sons, Inc., 1948.

2. Chestnut, Harold, and Mayer, Robert W.: Servomechanisms and Regulating Systems Design. Vol. I. John Wiley & Sons, Inc., 1951.

3. Otto, Edward W., and Taylor, Burt L., III: Dynamics of a Turbojet Engine Considered as a Quasi-Static System. NACA Rep. 1011, 1951. (Formerly NACA TN 2091.)

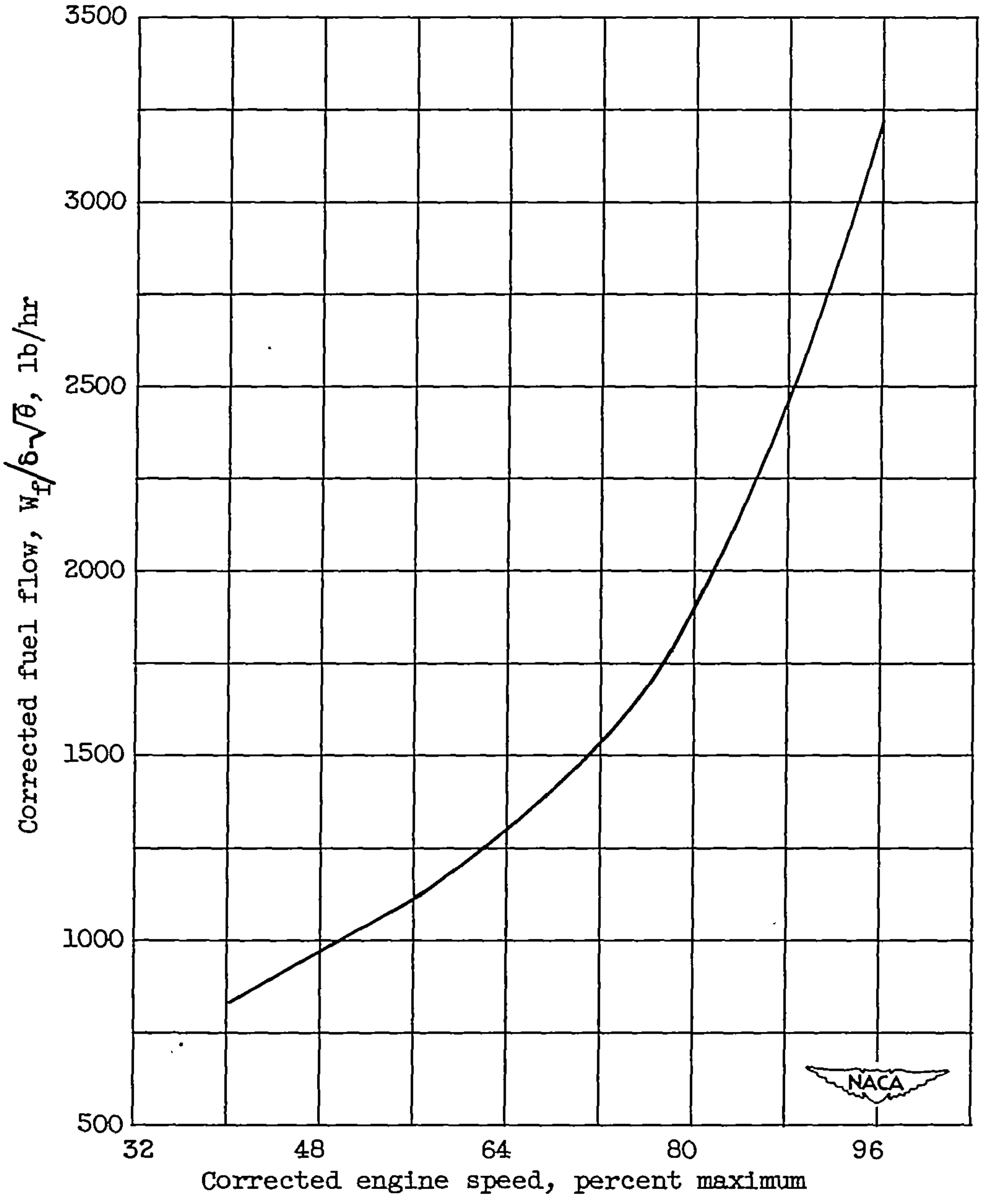
4. Himmel, Seymour C., and Krebs, Richard P.: The Effect of Changes in Altitude on the Controlled Behavior of a Gas-Turbine Engine. Jour. Aero. Sci., vol. 18, no. 7, July 1951, pp. 433-441.

5. LaVerne, Melvin E., and Boksenbom, Aaron S.: Frequency Response of Linear Systems from Transient Data. NACA Rep. 977, 1950. (Formerly NACA TN 1935.)

6. Delio, Gene J.: Evaluation of Three Methods for Determining Dynamic Characteristics of a Turbojet Engine. NACA TN 2634, 1952.

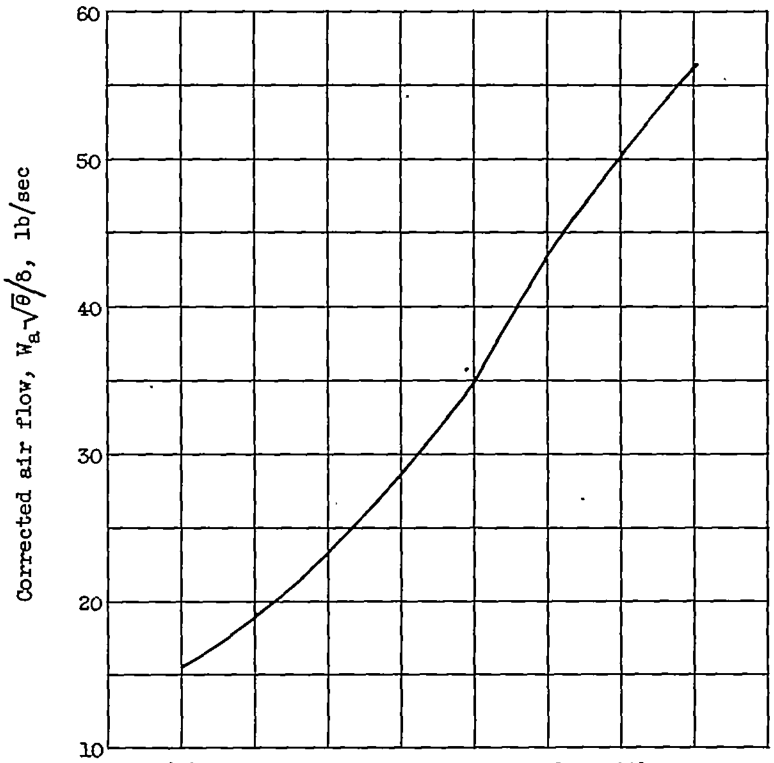
7. Shames, Harold, Himmel, Seymour C., and Blivas, Darnold: Frequency Response of Positive-Displacement Variable-Stroke Fuel Pump. NACA TN 2109, 1950.

8. Hall, Albert C.: The Analysis and Synthesis of Linear Servomechanisms. Chap. II. Technology Press, M.I.T. (Cambridge), 1943, pp. 19-27.

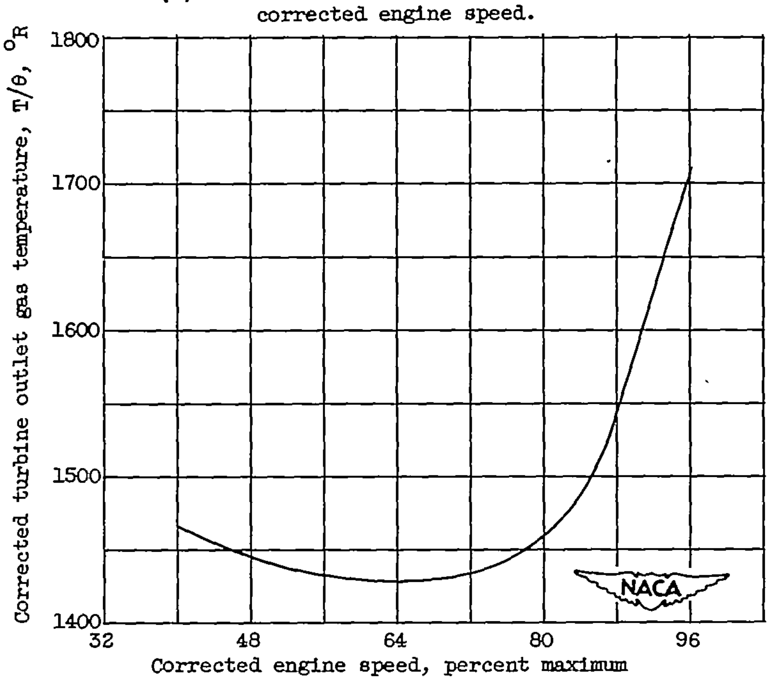


(a) Variation of corrected fuel flow with corrected engine speed.

Figure 1. - Steady-state performance curves of test engine.



(b) Variation of corrected air flow with corrected engine speed.



(c) Variation of corrected turbine outlet gas temperature with corrected engine speed.

Figure 1. - Concluded. Steady-state performance curves of test engine.

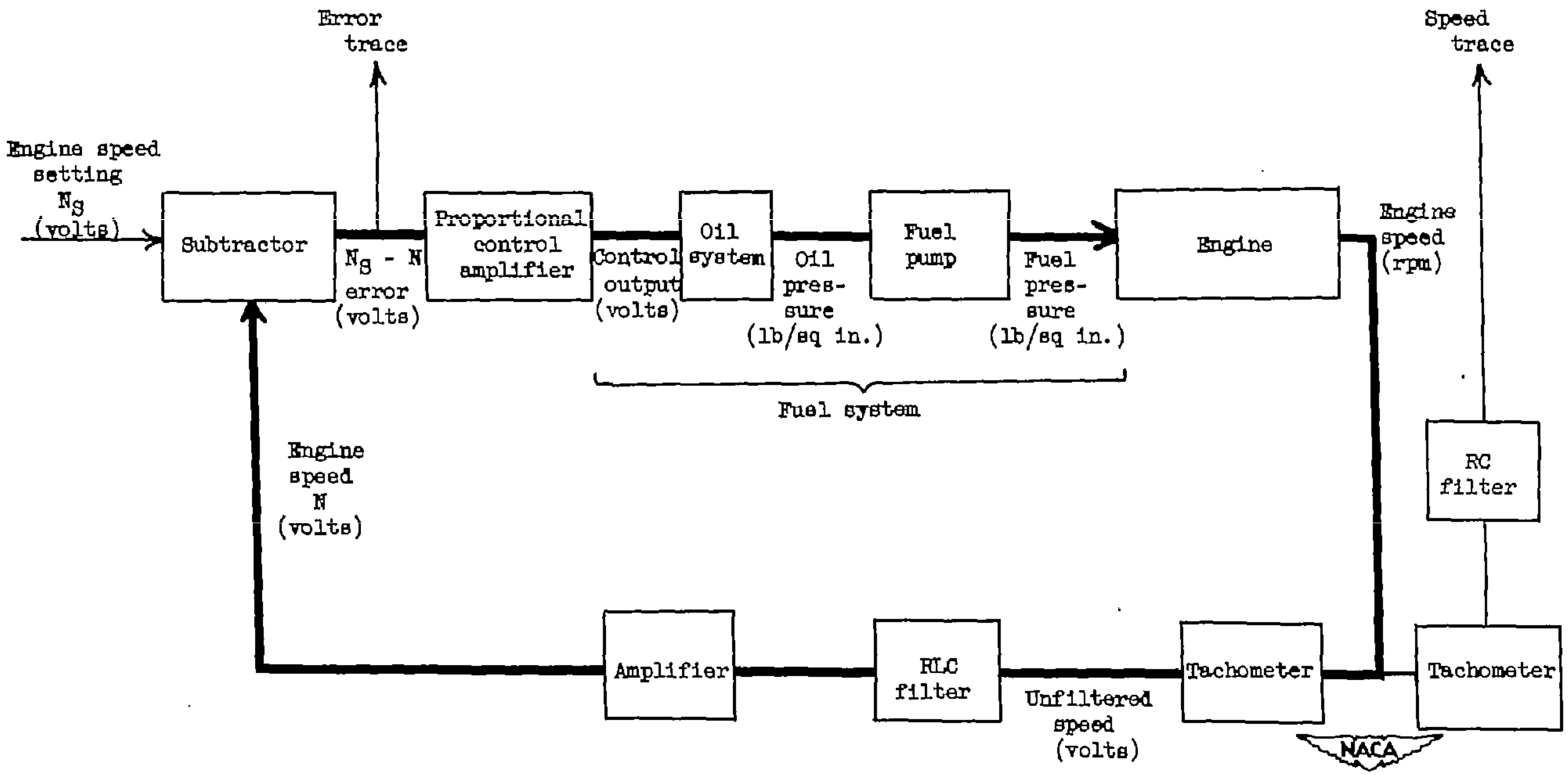
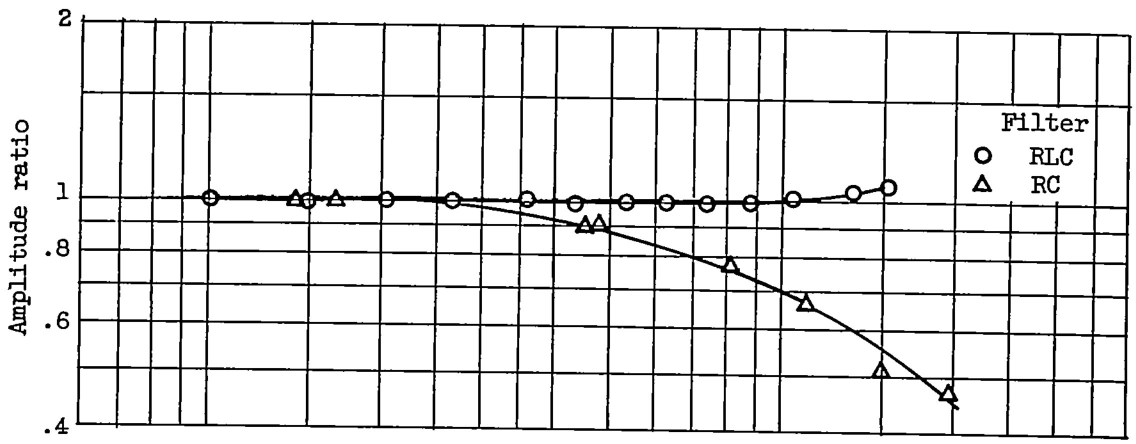
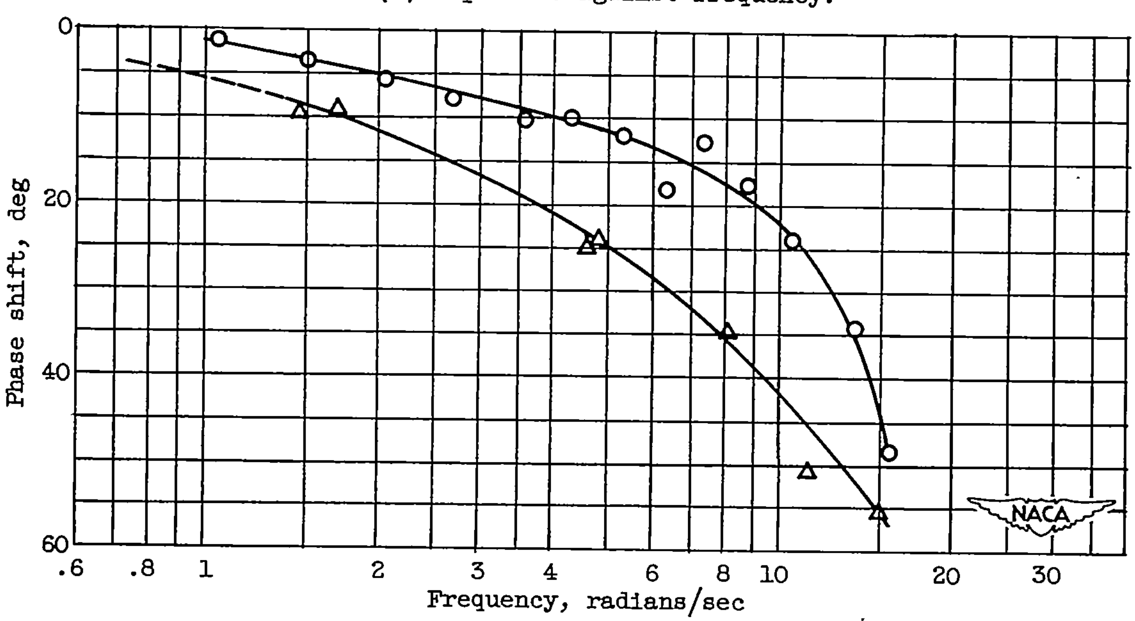


Figure 2. - Schematic diagram of closed loop controlled engine system.

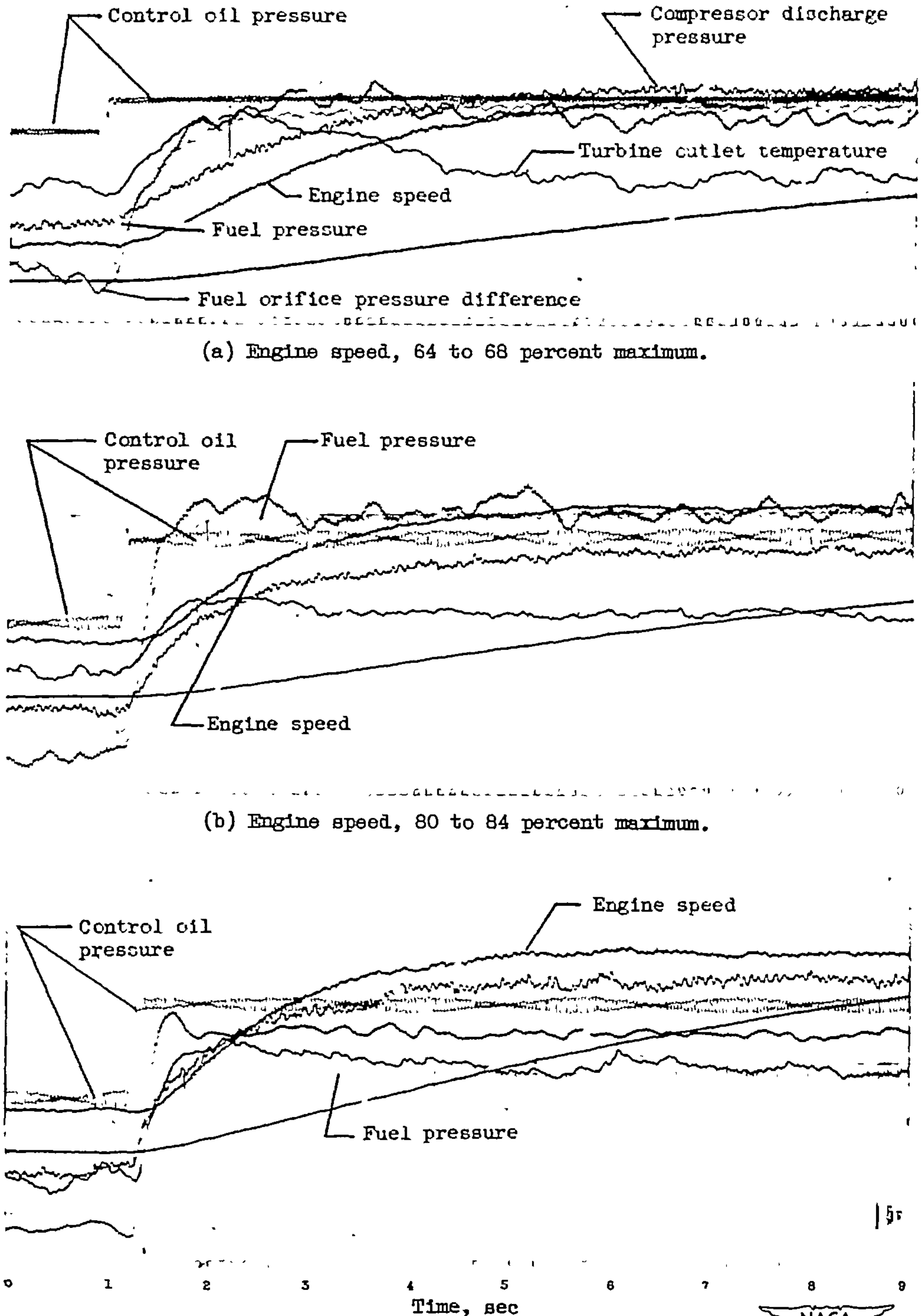


(a) Amplitude against frequency.



(b) Phase shift against frequency.

Figure 3. - Frequency response of filters.



(a) Engine speed, 64 to 68 percent maximum.

(b) Engine speed, 80 to 84 percent maximum.

(c) Engine speed, 88 to 92 percent maximum.

Figure 4. - Response of uncontrolled engine to approximate step in fuel flow.

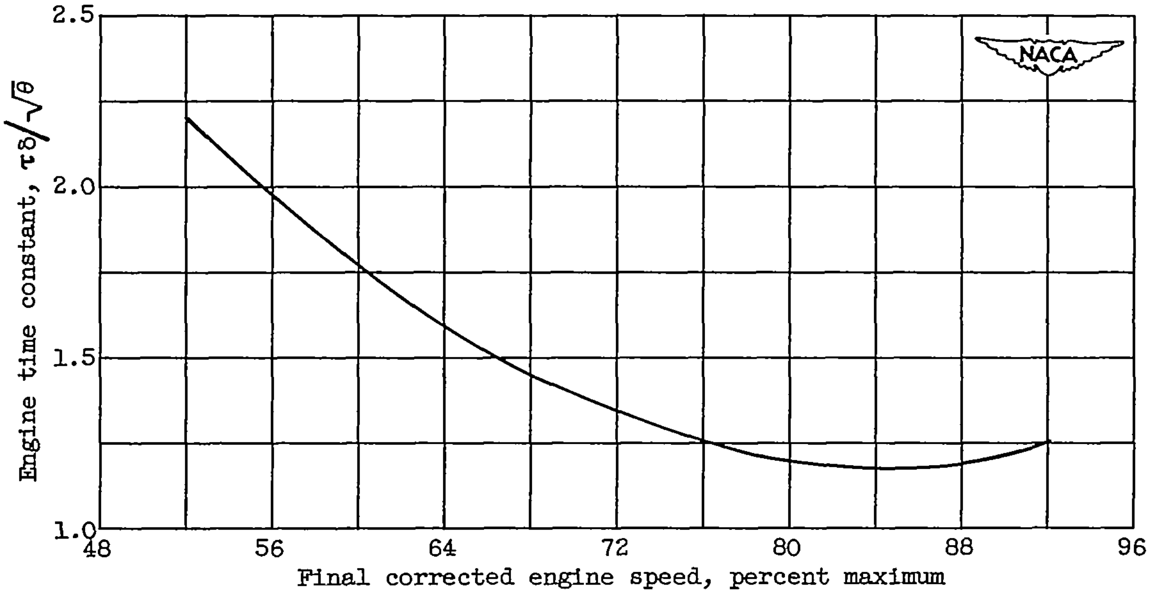
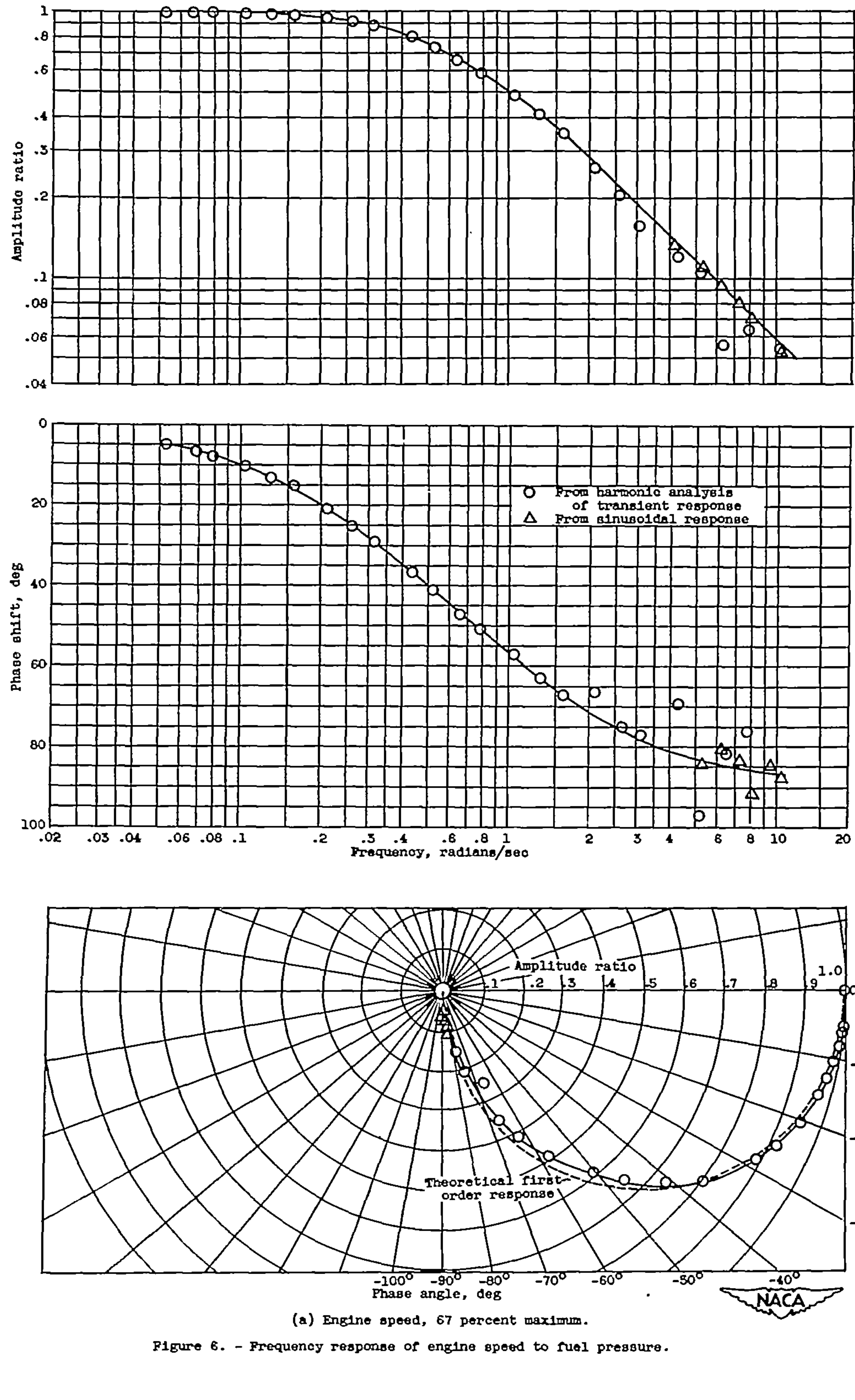
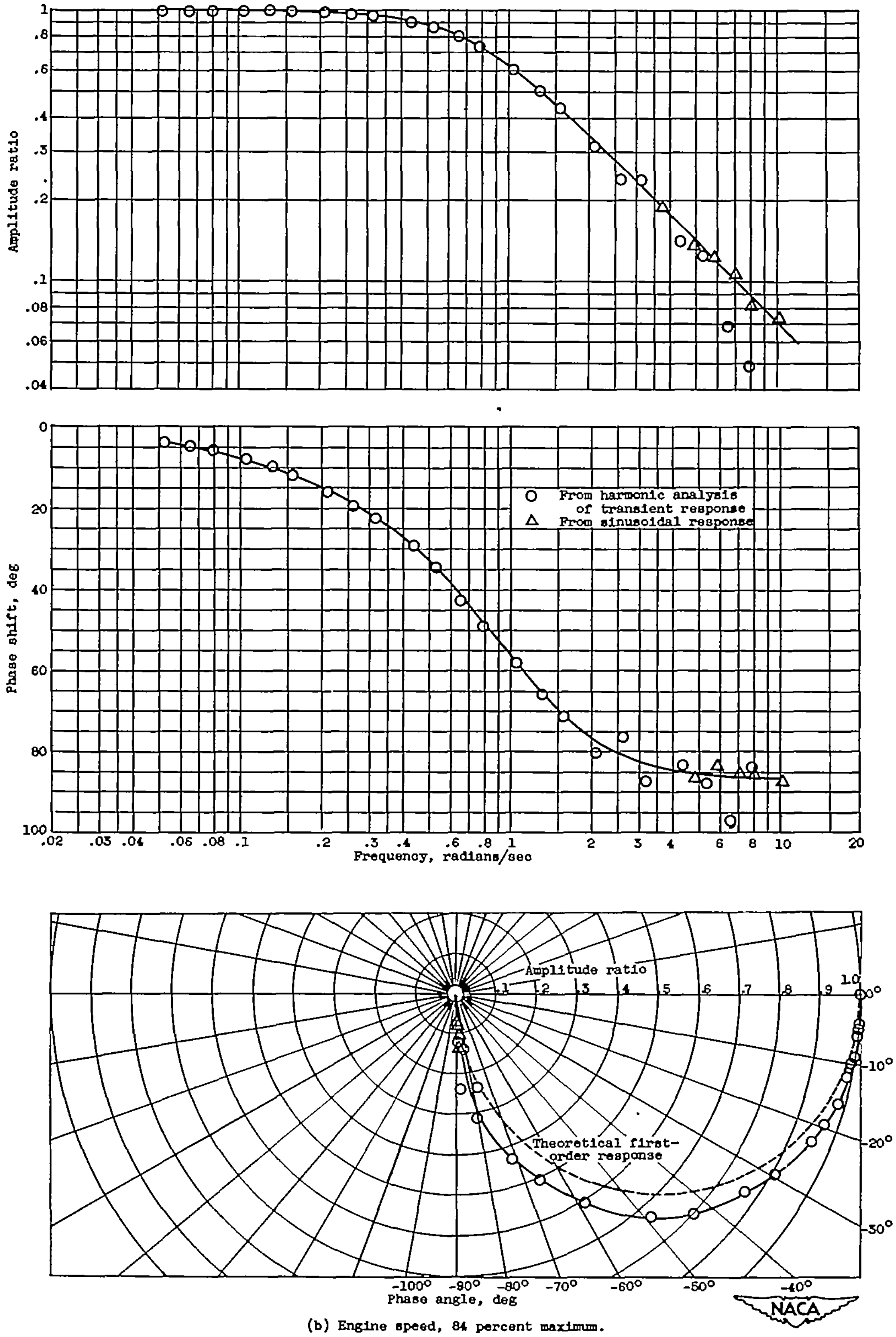


Figure 5. - Variation of engine time constant with engine speed.



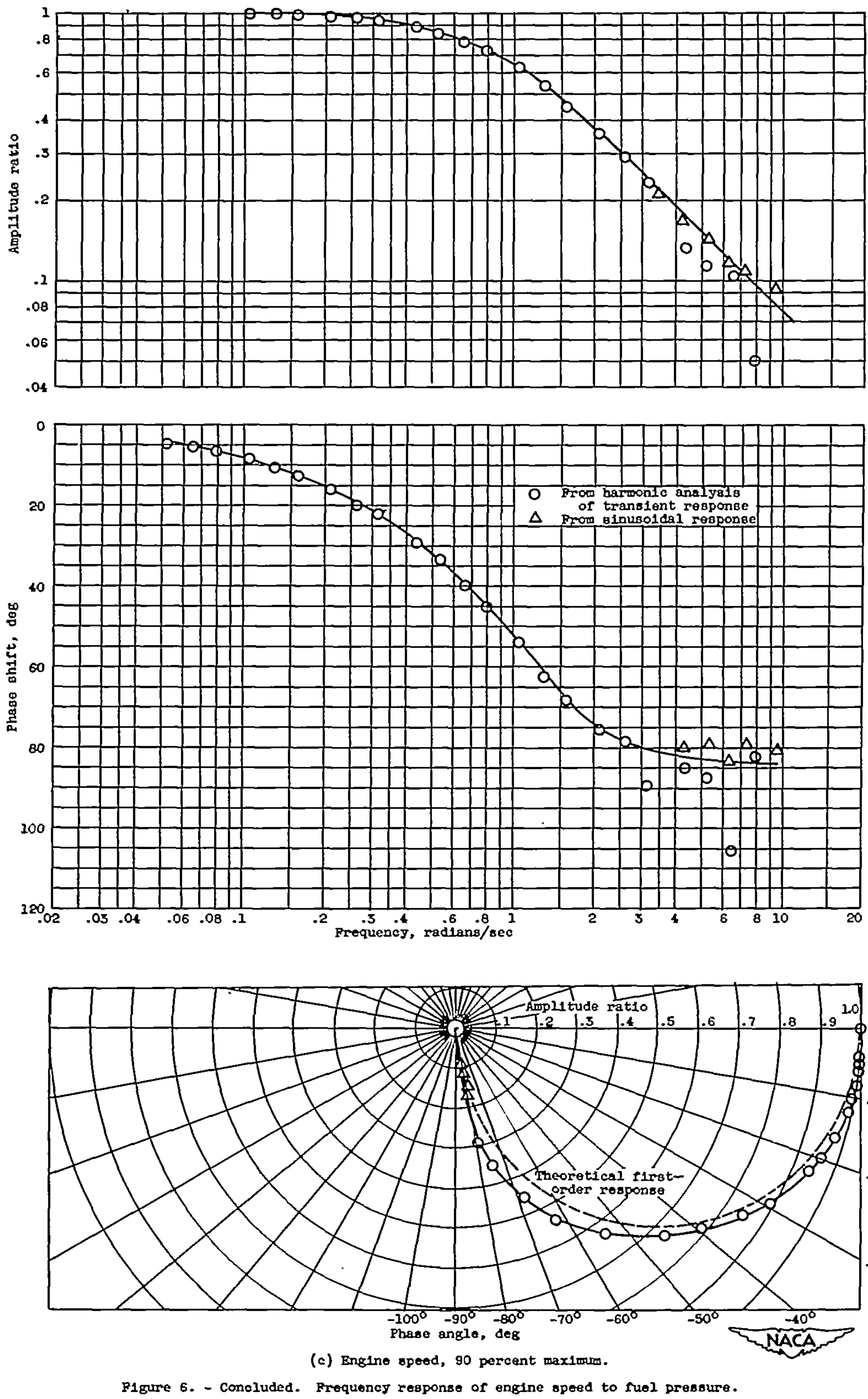
(a) Engine speed, 67 percent maximum.

Figure 6. - Frequency response of engine speed to fuel pressure.



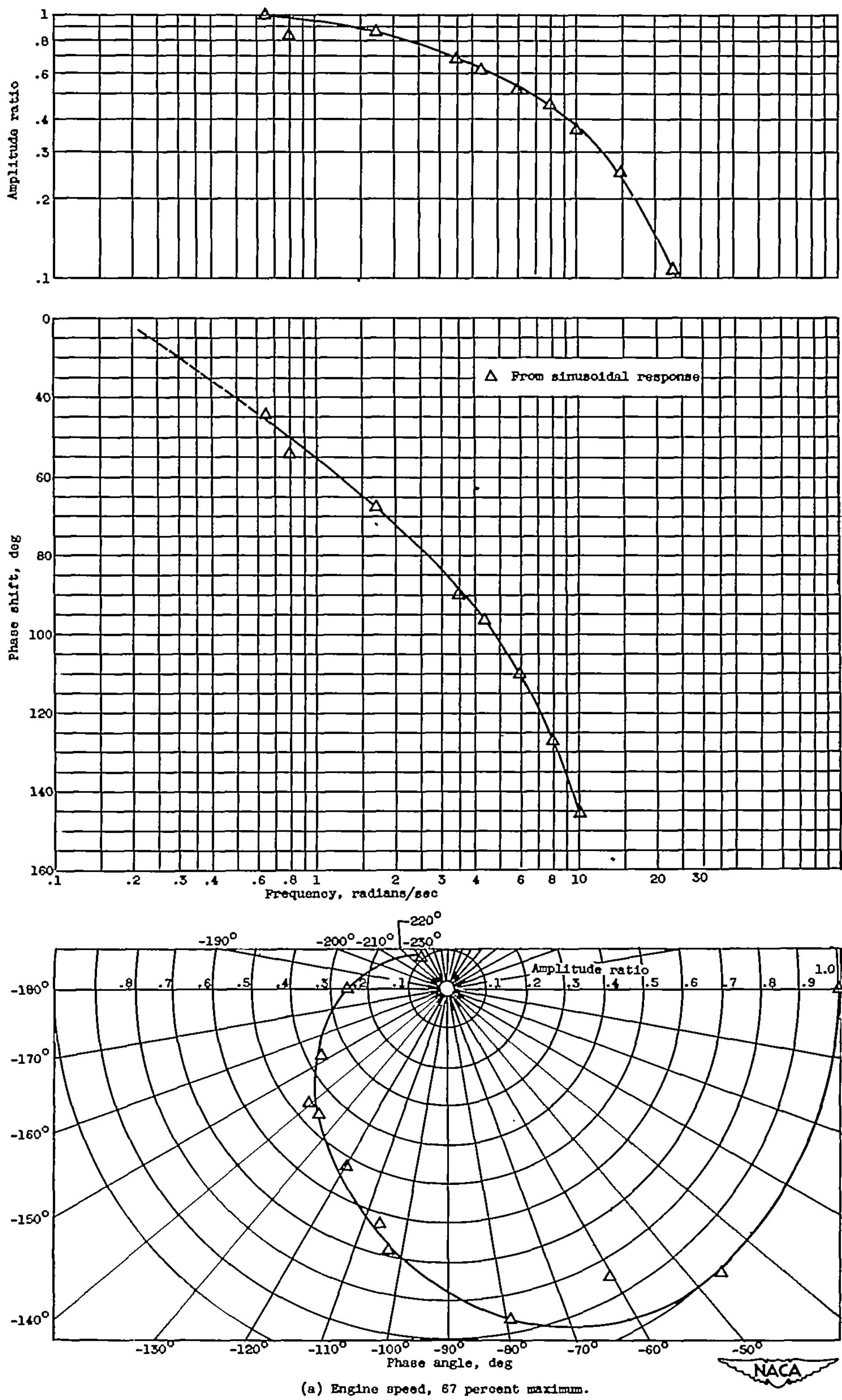
(b) Engine speed, 84 percent maximum.

Figure 6. - Continued. Frequency response of engine speed to fuel pressure.



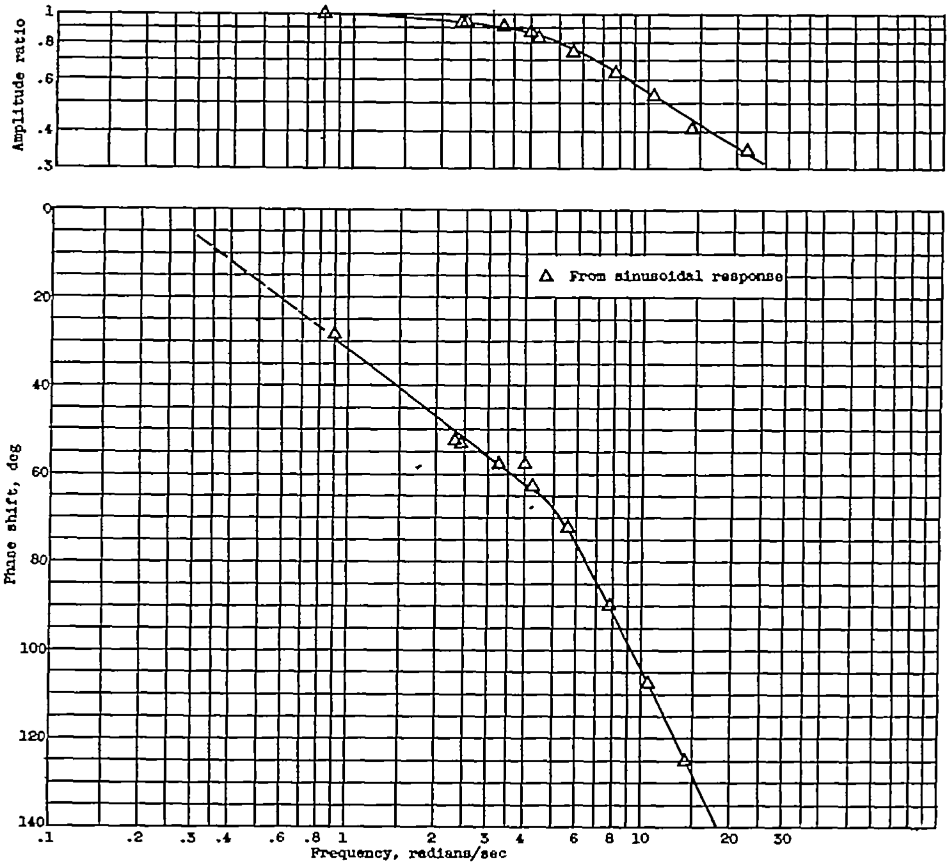
(c) Engine speed, 90 percent maximum.

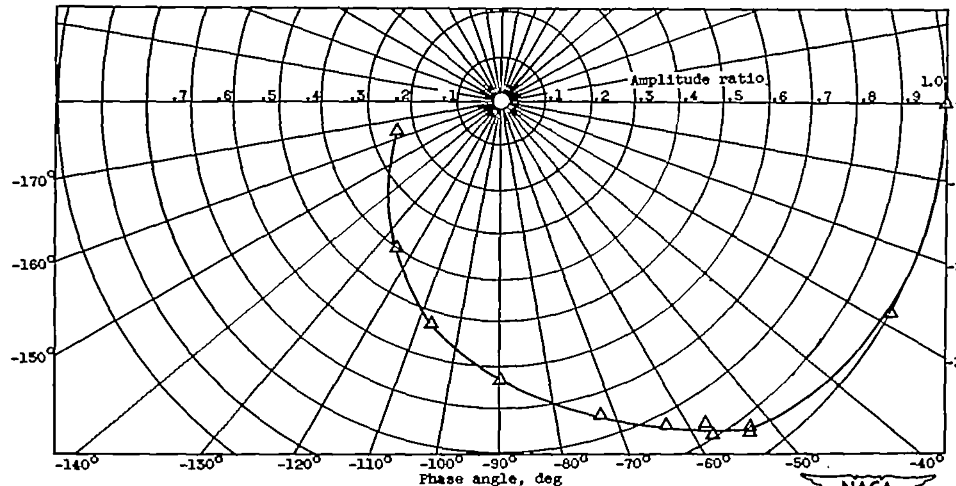
Figure 6. - Concluded. Frequency response of engine speed to fuel pressure.



(a) Engine speed, 67 percent maximum.

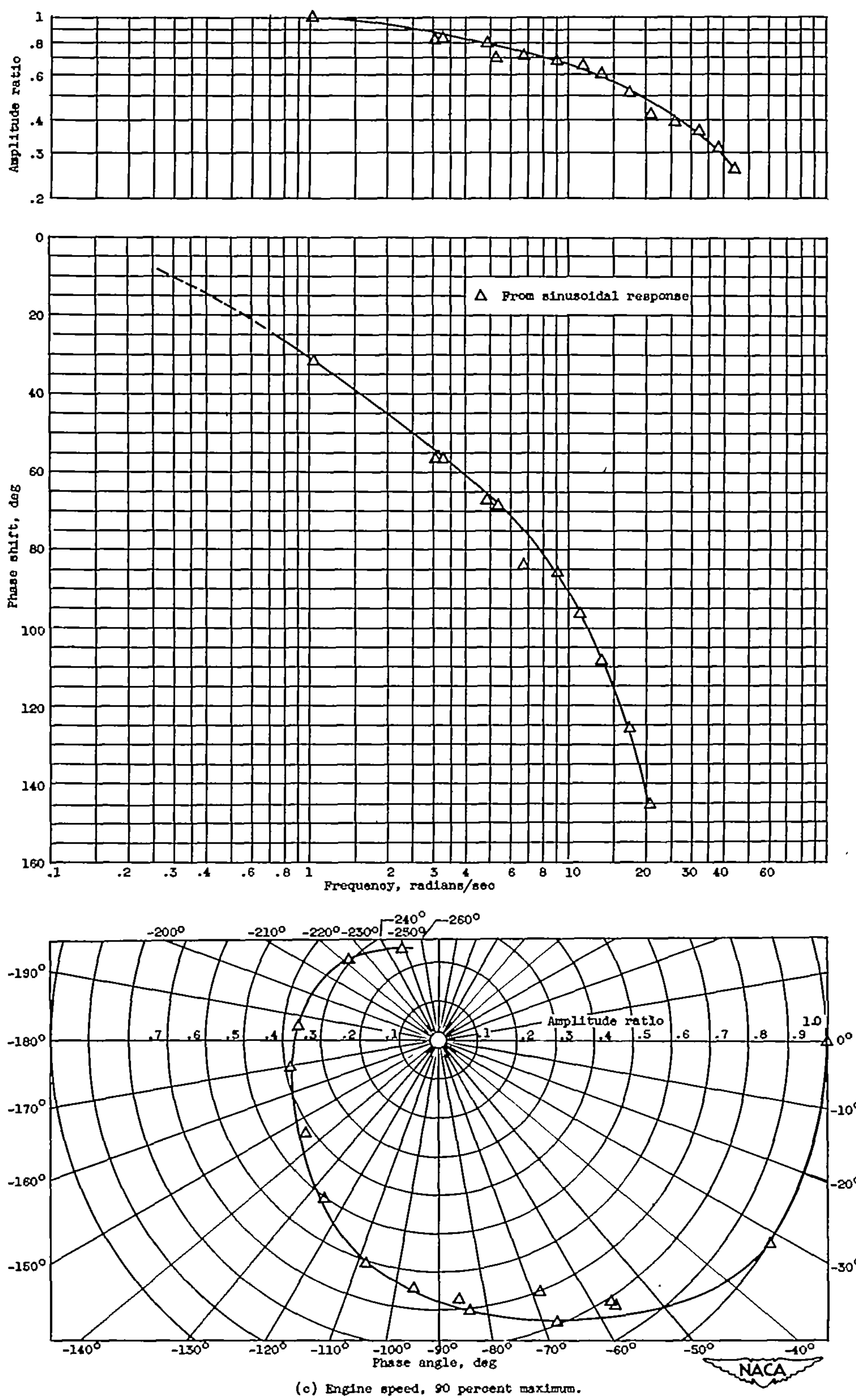
Figure 7. - Frequency response of fuel pressure to speed setting.





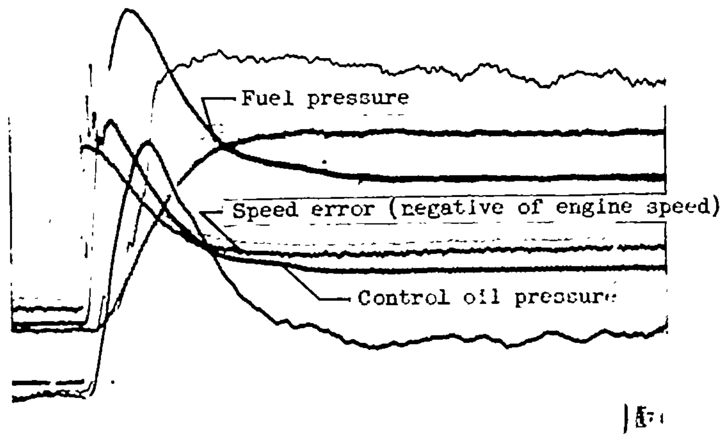
(b) Engine speed, 84 percent maximum.

Figure 7. - Continued. Frequency response of fuel pressure to speed setting.

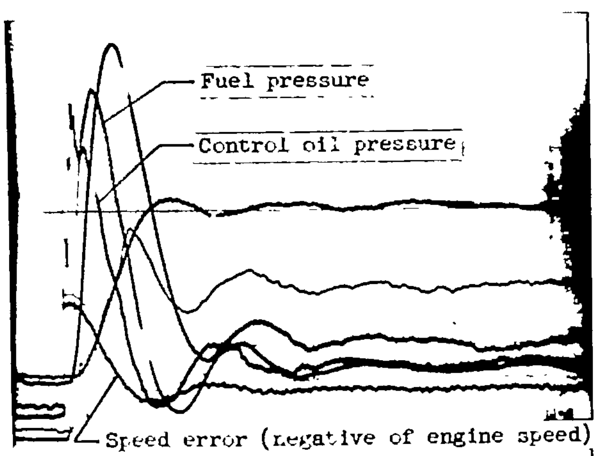


(c) Engine speed, 90 percent maximum.

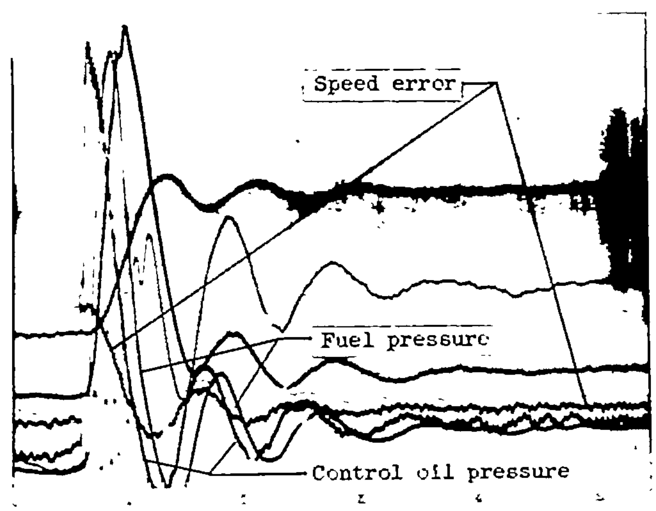
Figure 7. - Concluded. Frequency response of fuel pressure to speed setting.



(a) Loop gain, 2.



(b) Loop gain, 5.7.



Time, sec

(c) Loop gain, 10.

Figure 8. - Response of controlled engine to step change in speed setting. Final engine speed, 84 percent maximum.

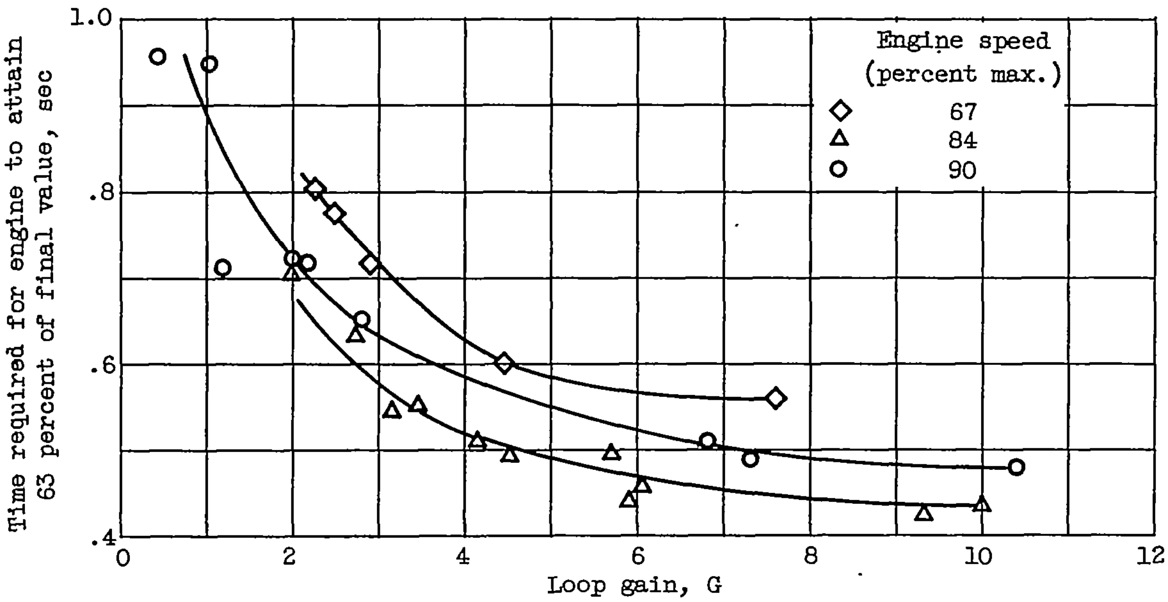


Figure 9. - Effect of loop gain on response time.

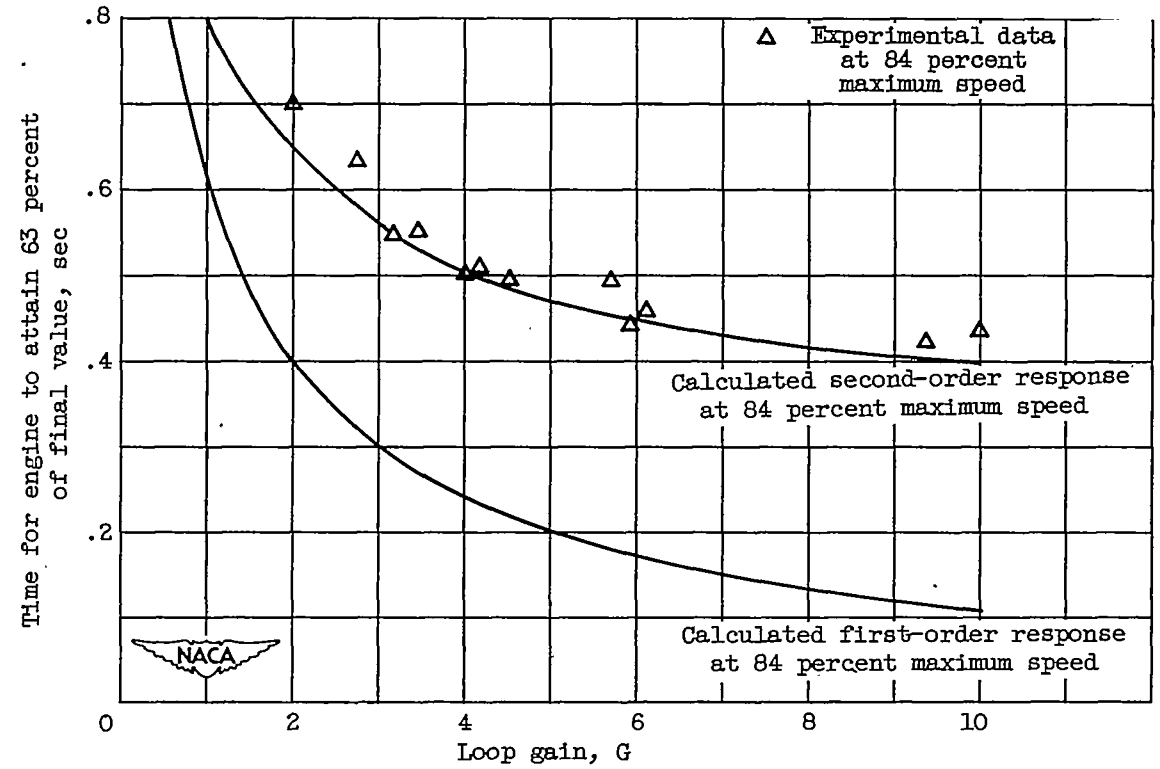


Figure 10. - Comparison of calculated response with experimental response for various loop gains.

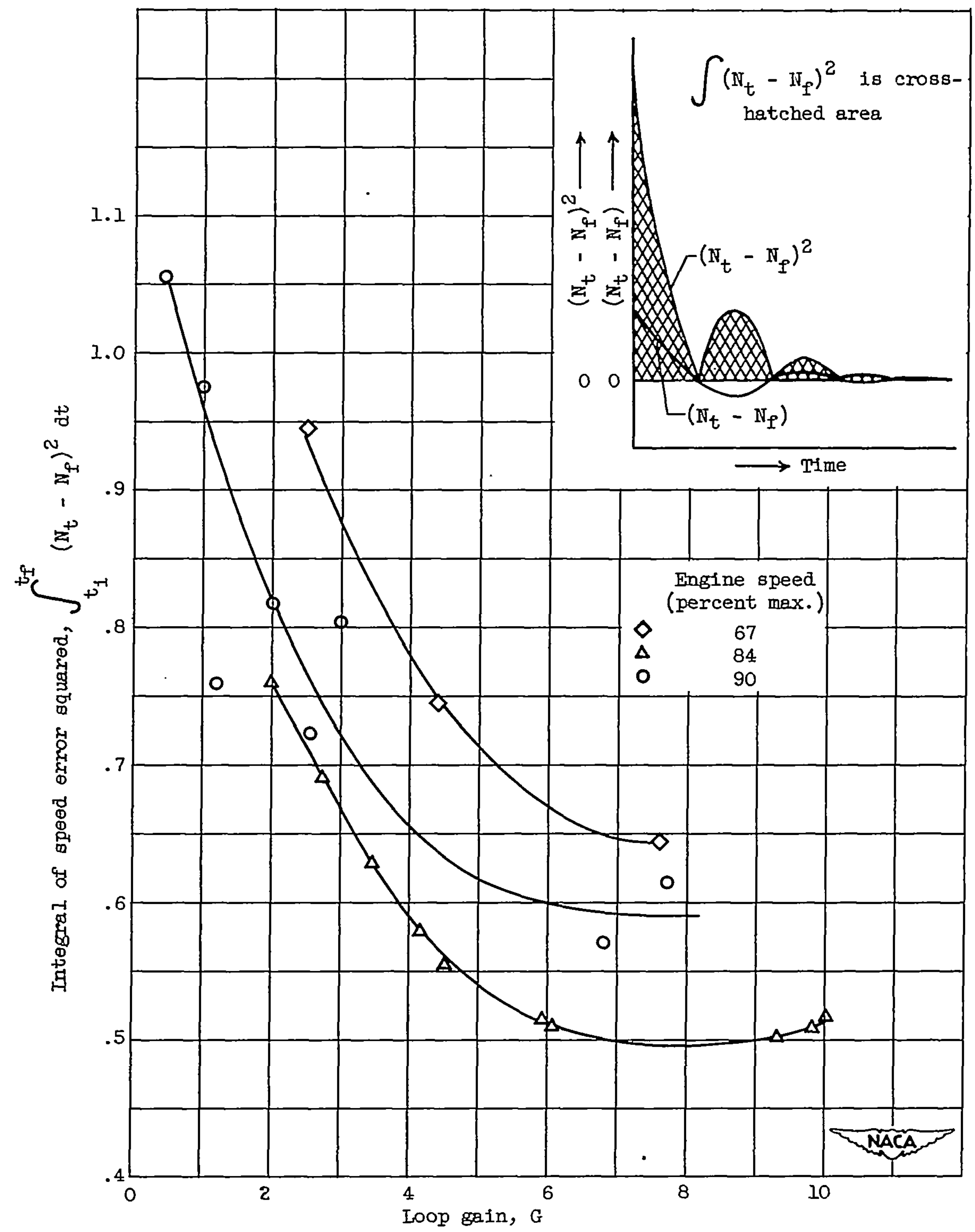
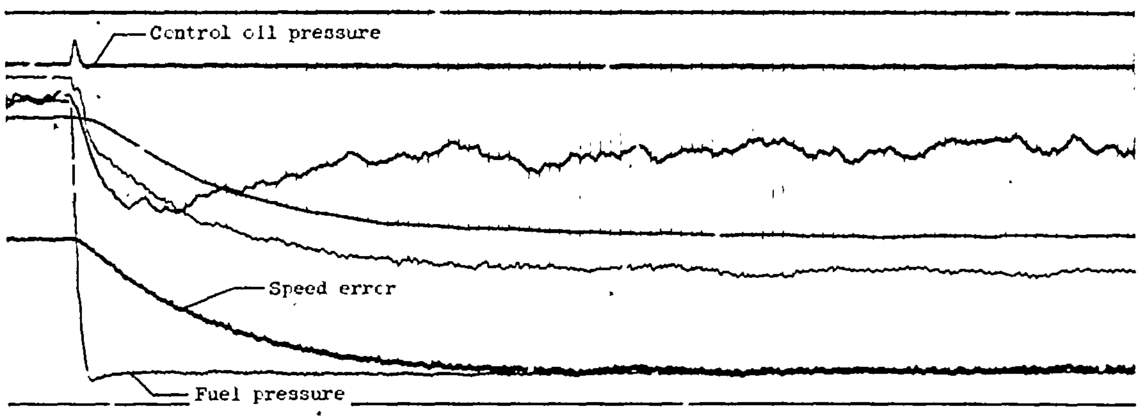
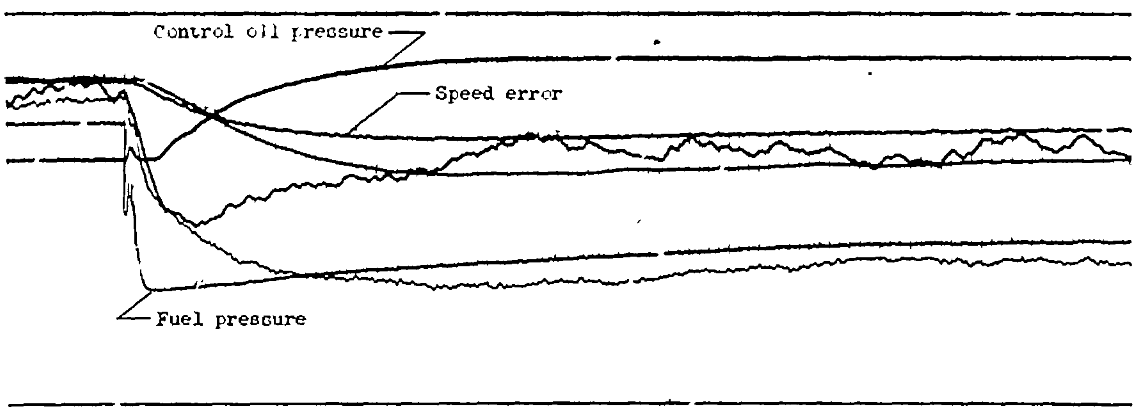


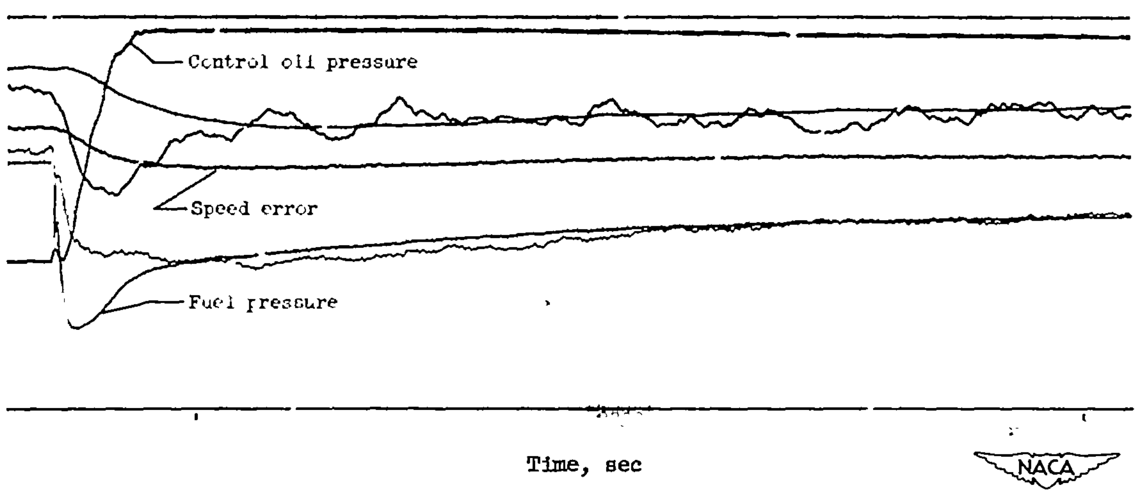
Figure 11. - Integral of speed error squared as response criterion for various loop gains.



(a) Loop gain, 0 (Uncontrolled engine).



(b) Loop gain, 0.51.



(c) Loop gain, 2.4.

Figure 12. - Response of controlled engine to fuel flow disturbance.

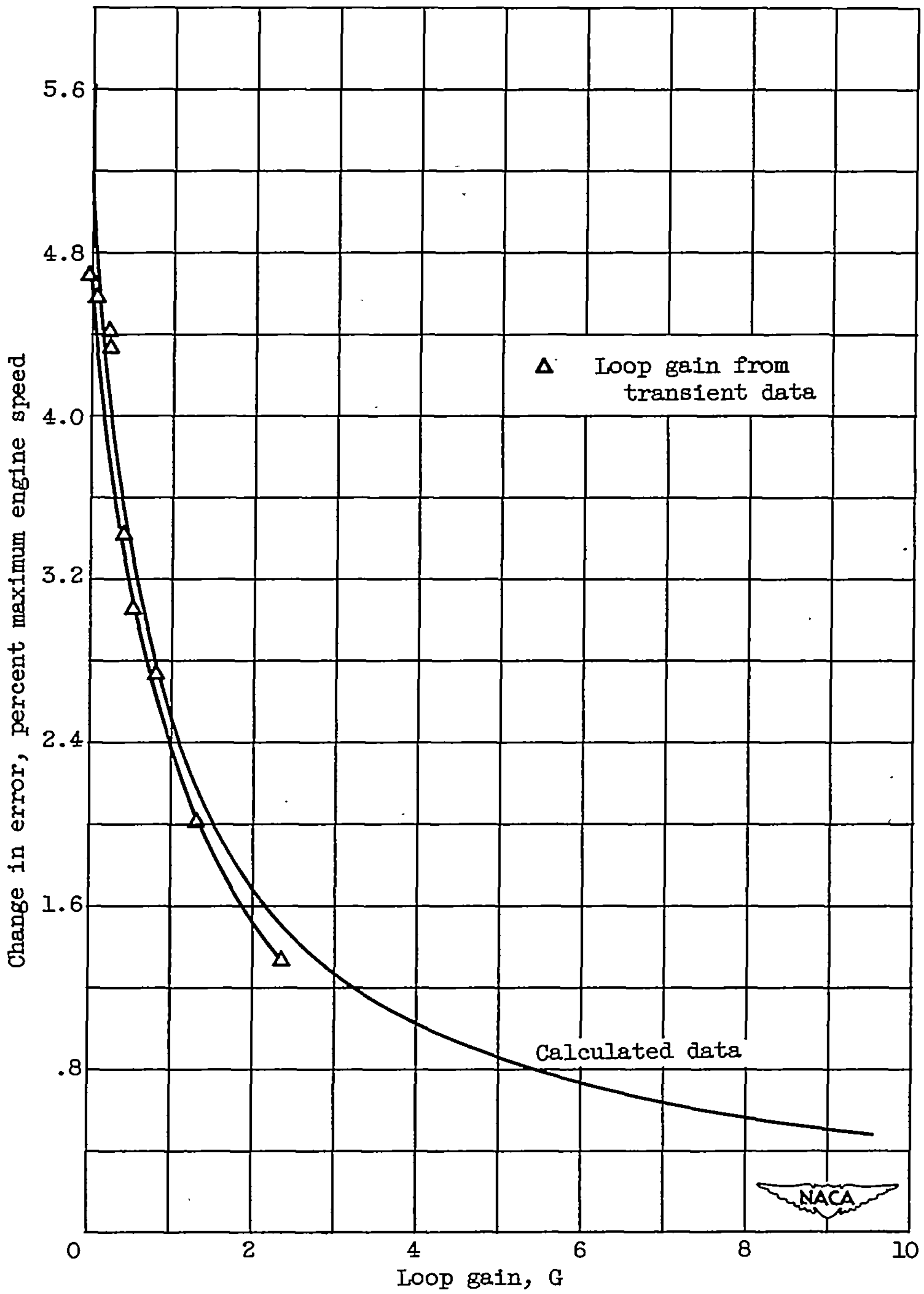
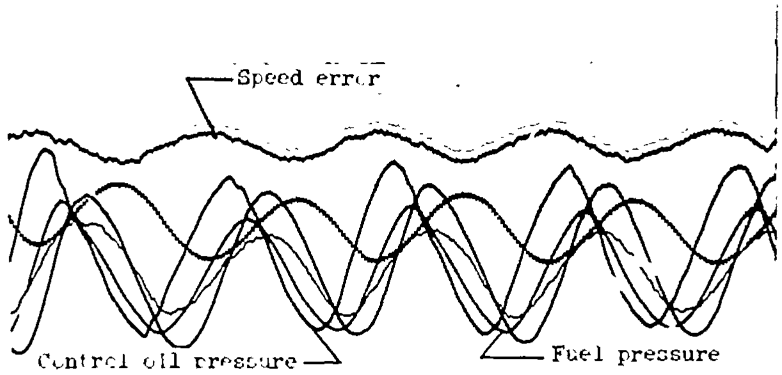
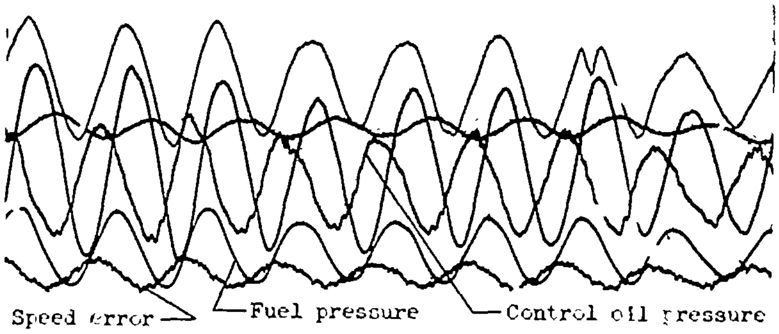


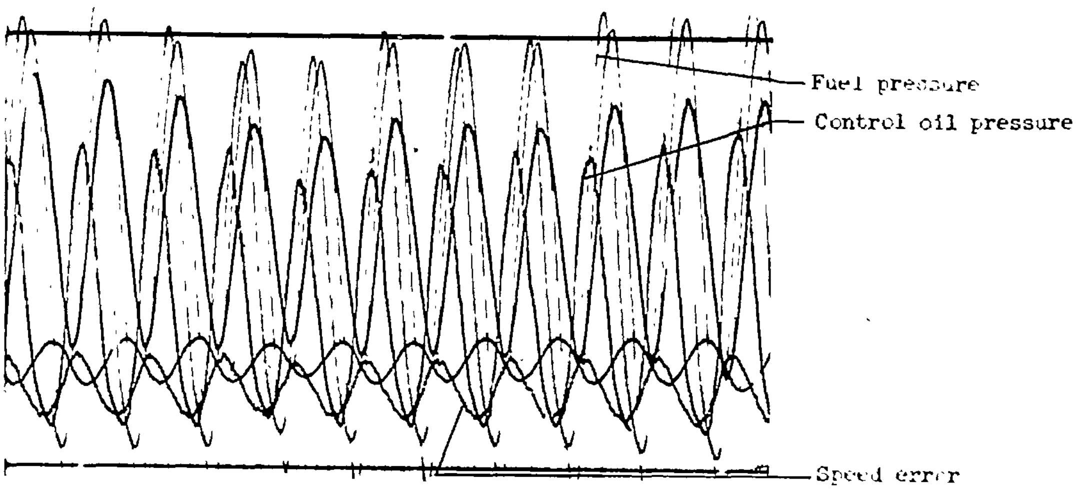
Figure 13. - Variation of speed error with loop gain following step disturbance in fuel system equivalent to 5 percent maximum engine speed.



(a) Engine speed, 64 percent maximum.

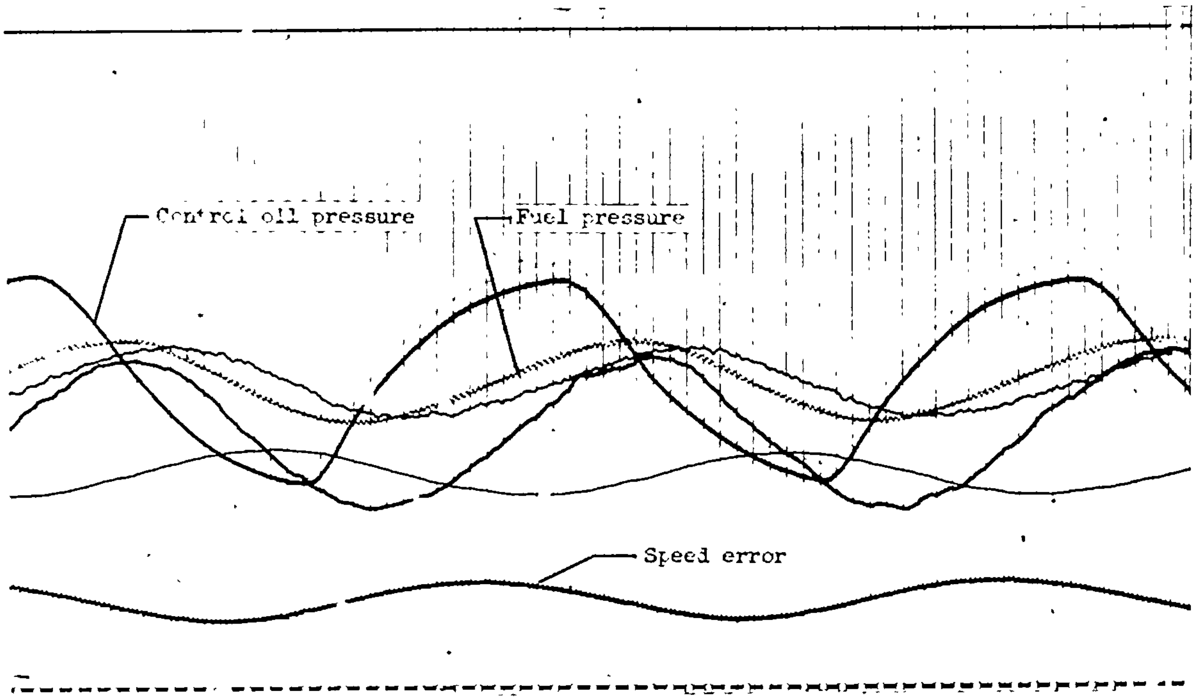


(b) Engine speed, 80 percent maximum.

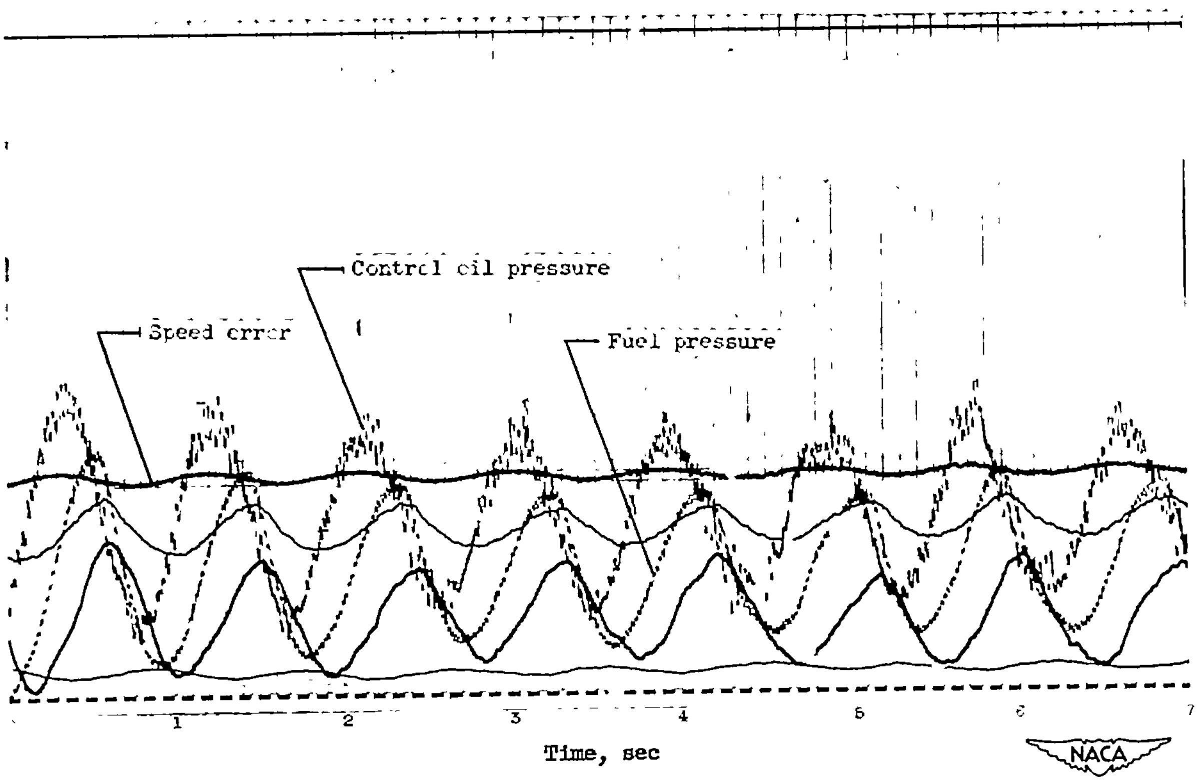


(c) Engine speed, 88 percent maximum.

Figure 14. - Controlled engine instability.



(a) Engine speed, 64 percent maximum.



(b) Engine speed, 88 percent maximum.

Figure 15. - Controlled engine instability. System revised by increased phase shift.

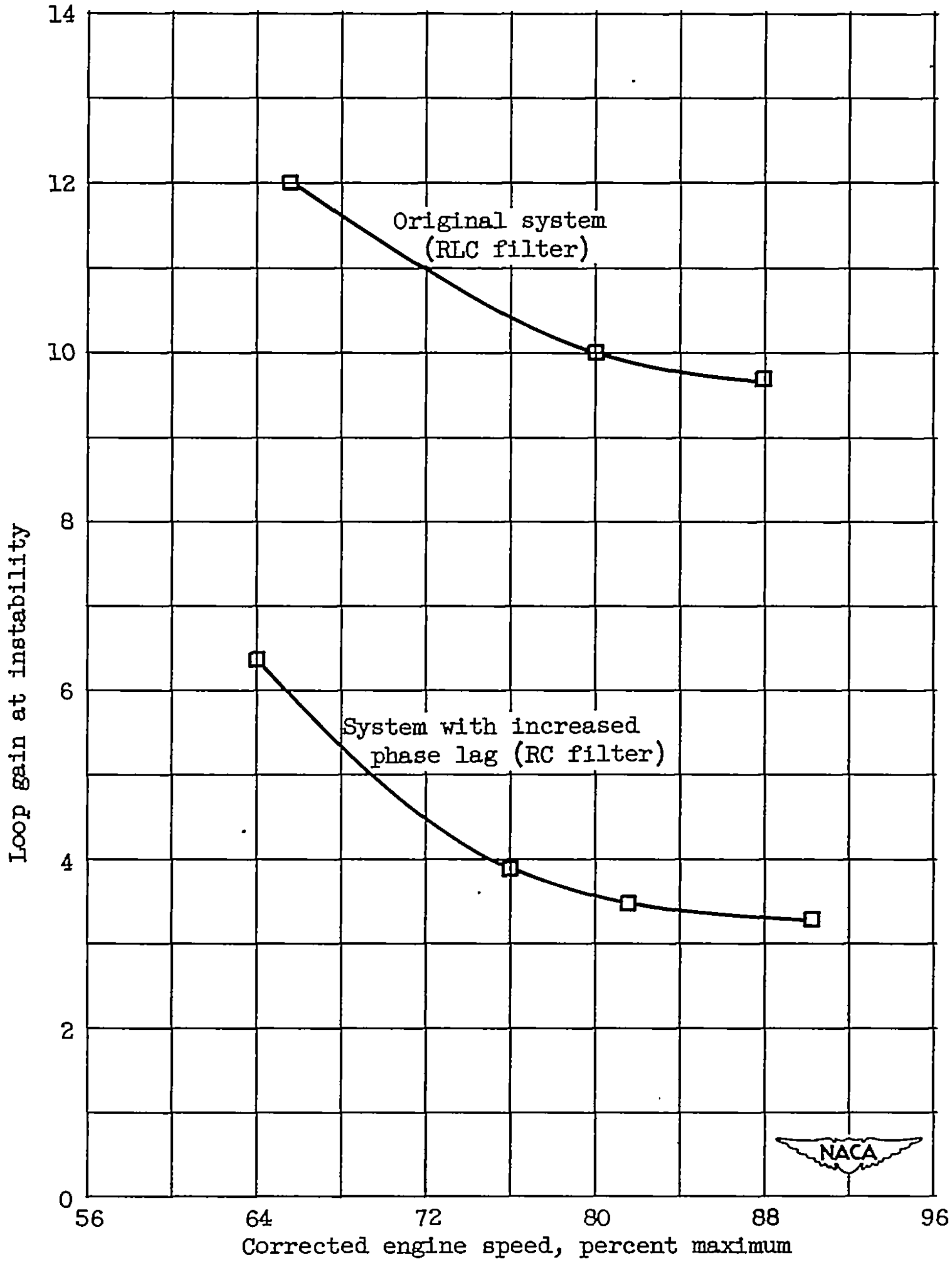
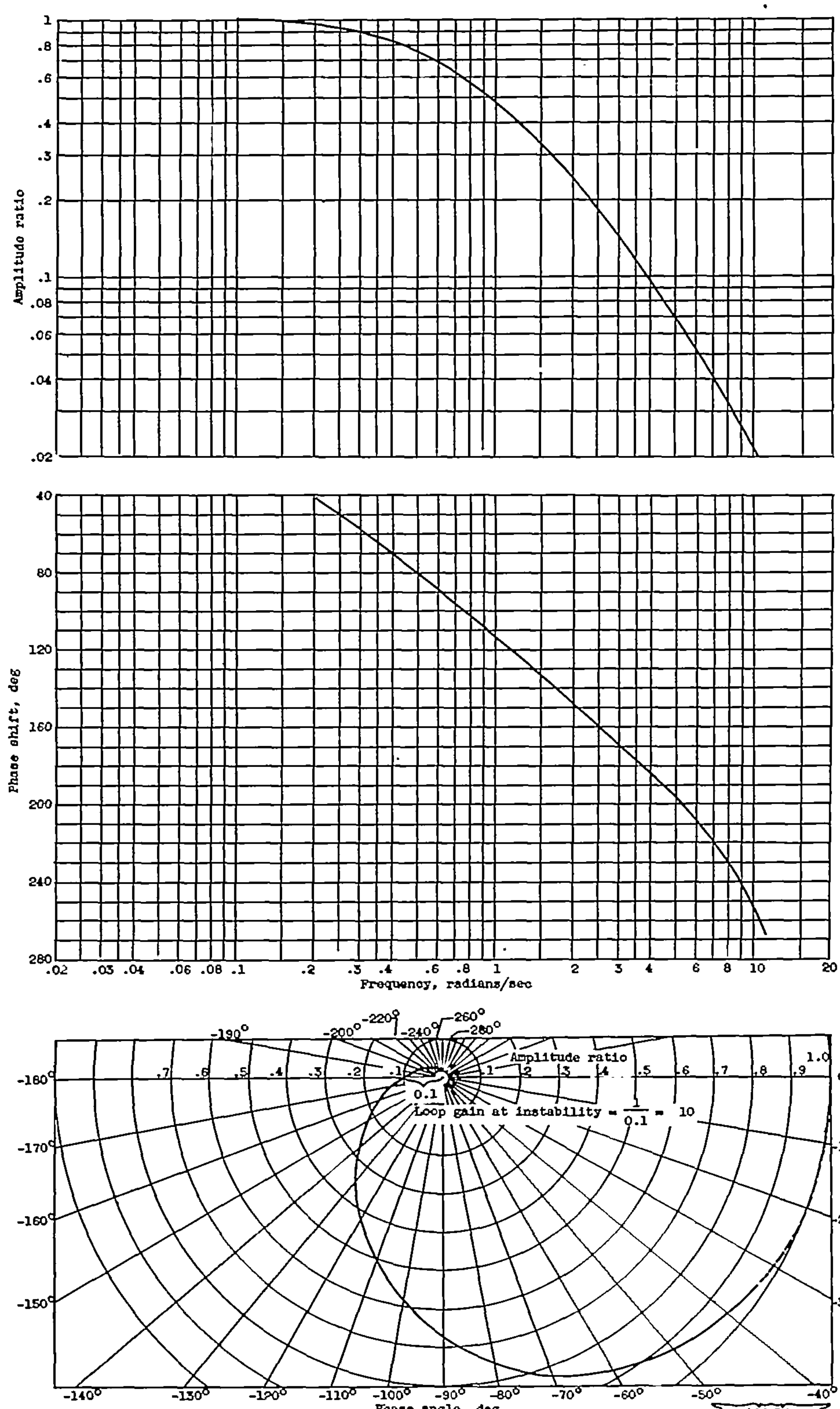
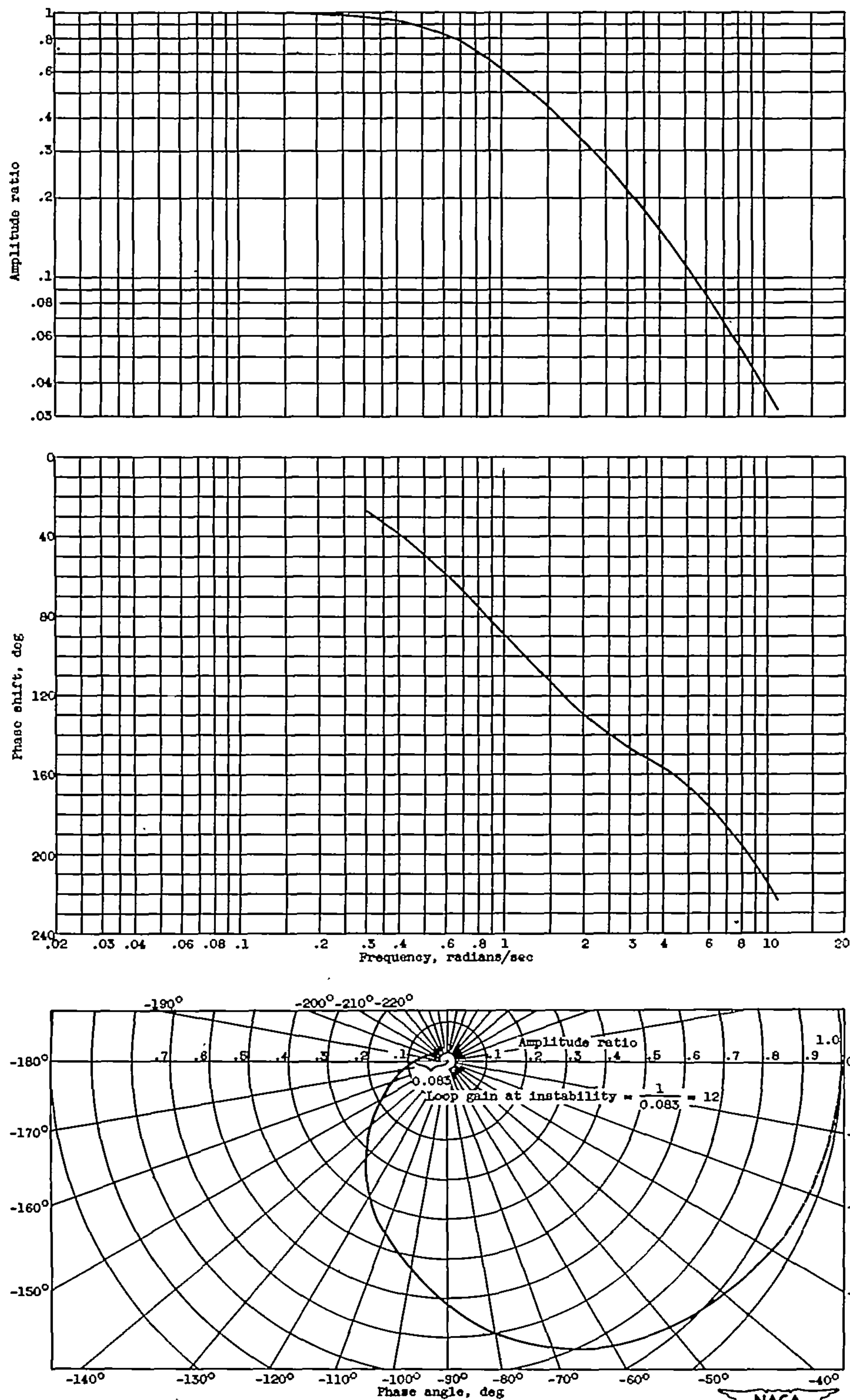


Figure 16. - Loop gain at instability for various engine speeds.



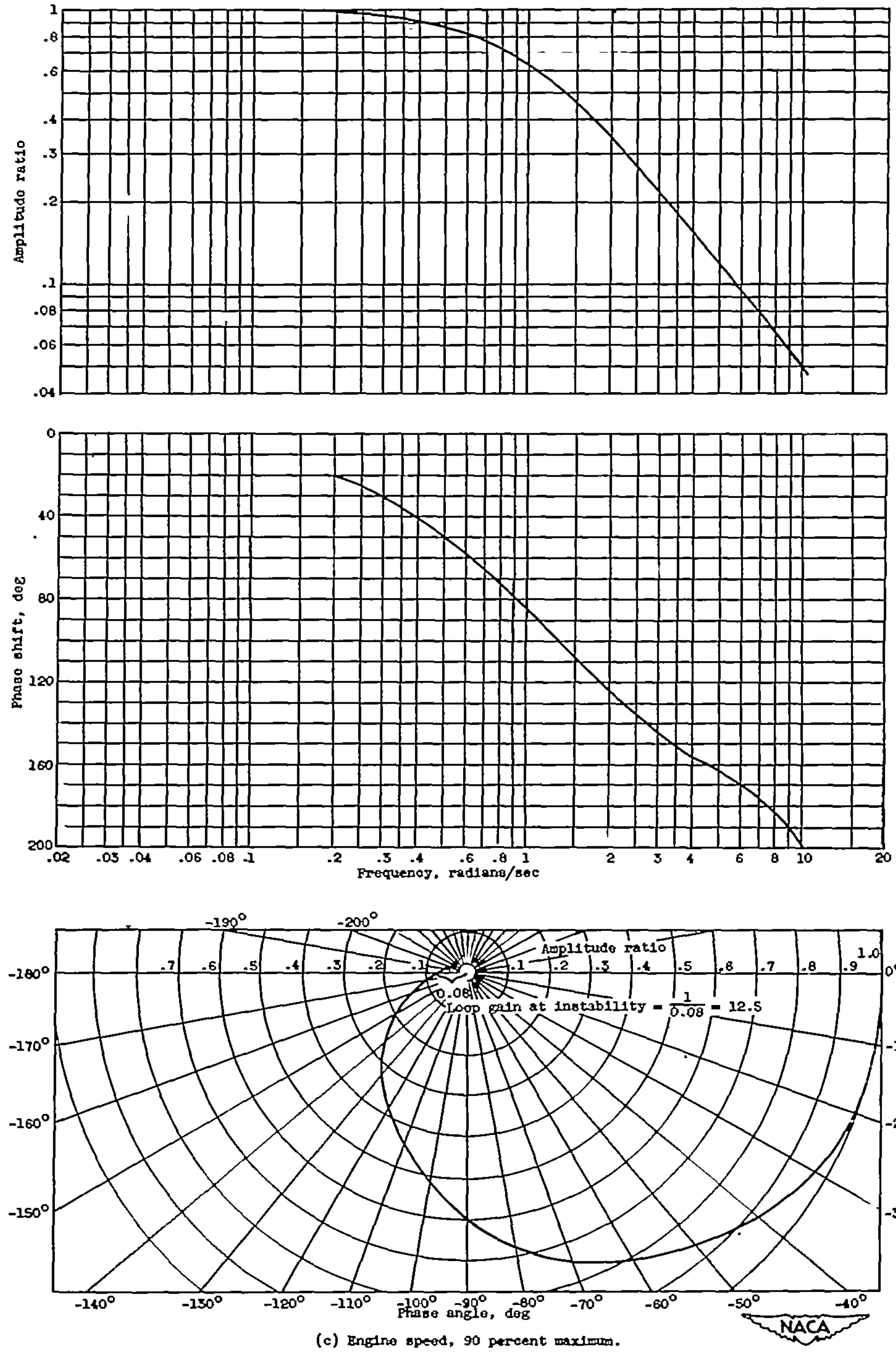
(a) Engine speed, 67 percent maximum.

Figure 17. - Frequency response of all loop components in cascade.



(b) Engine speed, 84 percent maximum.

Figure 17. - Continued. Frequency response of all loop components in cascade.



(c) Engine speed, 90 percent maximum.

Figure 17. - Concluded. Frequency response of all loop components in cascade.

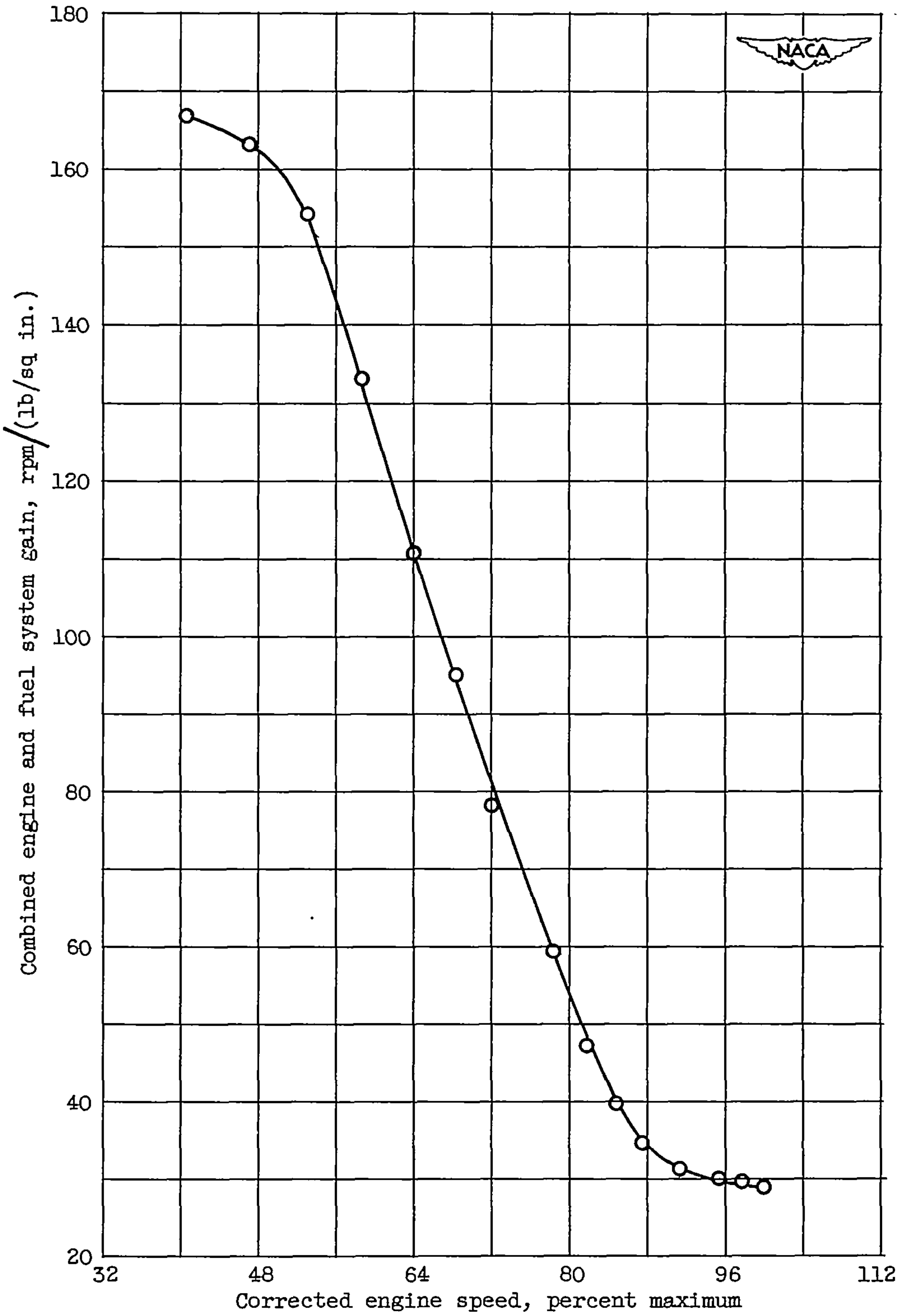


Figure 18. - Variation of combined engine and fuel system gain with engine speed.

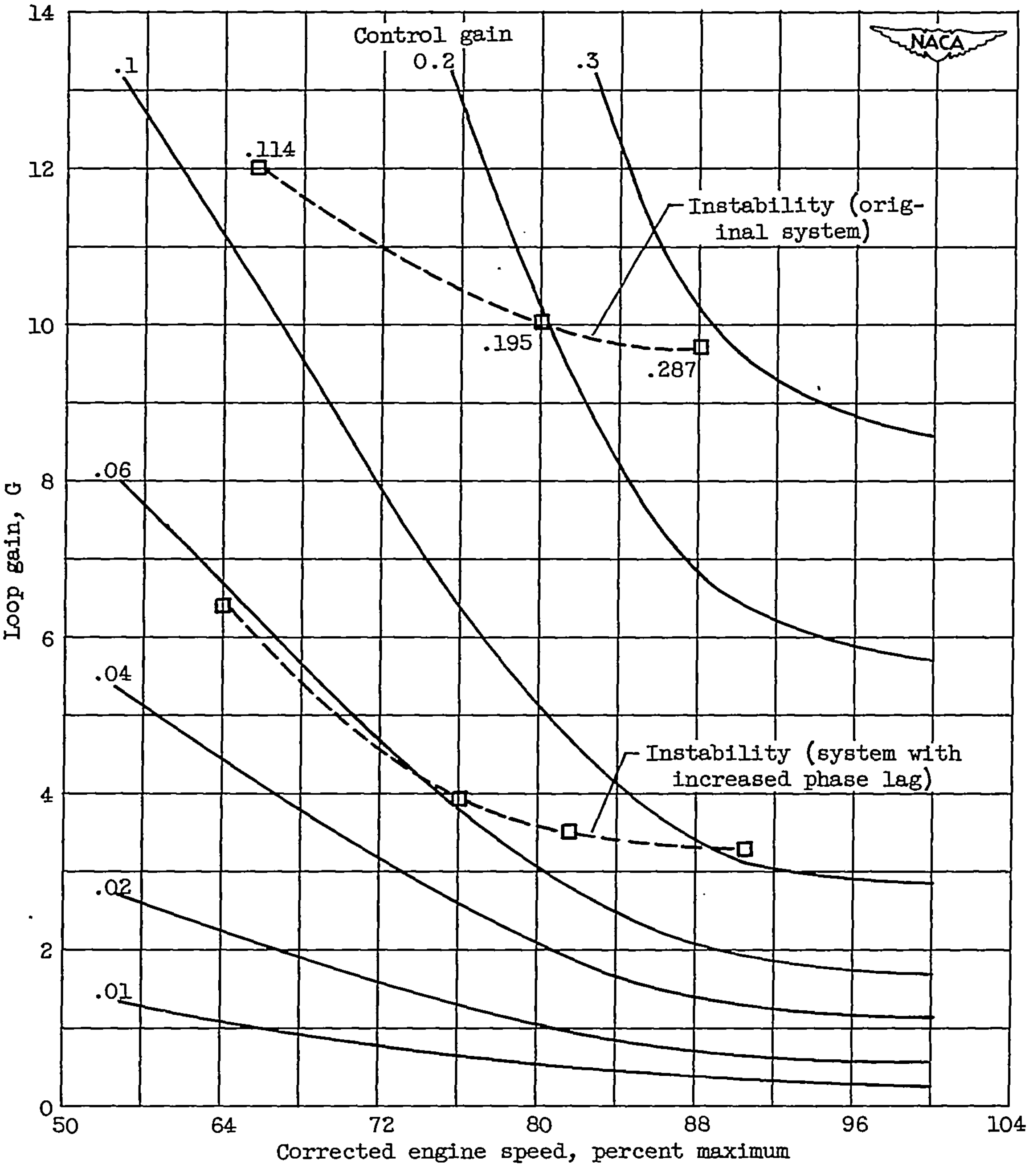


Figure 19. - Variation of loop gain with engine speed for constant values of control gain $= \dfrac{\text{Loop gain}}{\text{Engine gain} \times \text{Fuel system gain}}$.